the output will be

$$xo = A_1 \sin (wt + \phi_1) + A_3 \sin (3wt\phi_3) \tag{2}$$

where ϕ_1 is the phase shift at w, ϕ_3 is the phase shift at $3w$, A_1 = gain at w, and A_3 = gain at $3w$. The output signal is delayed by ϕ_1/w so changing the time origin of the signal

$$wt + \phi_1 = w\tau$$

or

$$t = \tau - \phi_1/w,$$

$$xo = A_1 \sin w\tau + A_3 \sin \{3w(\tau - \phi_1/w) + \phi_3\} \tag{3}$$

$$xo = A_1 \sin w\tau + A_3 \sin \{3w\tau + \phi_3 - 3\phi_1\} \tag{4}$$

$$xo = A_1 \sin w\tau + A_3 \sin (3w\tau - \theta) \tag{5}$$

where the relative phase angle

$$\theta = 3\phi_1 - \phi_3. \tag{6}$$

For a system which has a region with a linear phase response where $\theta = 0$, a difference equation may be used to evaluate $\phi(w)$ from θ, i.e.,

$$3\phi(w) - \phi(3w) = \theta(w)$$

$$\phi(3w) - \tfrac{1}{3}\phi(9w) = \tfrac{1}{3}\theta(3w)$$

$$\tfrac{1}{3}\phi(9w) - \tfrac{1}{9}\phi(27w) = \tfrac{1}{9}\theta(9w).$$

Summing

$$3\phi(w) = \theta(w) + \tfrac{1}{3}\theta(3w) + \tfrac{1}{9}\theta(9w)$$

$$+ \text{higher terms which tend to } 0,$$

so

$$\phi(w) = \sum_{n=0}^{\infty} \frac{1}{3^{n+1}} \theta(3^n w).$$

As this series converges in the area of linear phase response, $\phi(w)$ may be evaluated, although terms independent of frequency will not be represented.

ACKNOWLEDGMENT

We would like to thank Dr. B. H. Brown of the Department of Medical Physics, Royal Hallamshire Hospital, Sheffield, for his initial encouragement and Dr. D. Woollons and Dr. M. English of the Department of Biomedical Engineering, University of Sussex, for advice and support. Also, Prof. J. M. Nightingale of the University of Southampton, for assistance with the mathematics in the Appendix.

REFERENCES

[1] P. Lynch, H. Dargie, S. Krickler, and D. Krickler, "Objective assessment of anti-anginal treatment: A double blind comparison of propranolol, nifedipine, and their combination," *Brit. Med. J.*, pp. 184-187, July 1980.

[2] V. Balasubramanian, A. Lahiri, H. L. Green, and F. D. Stott, "Ambulatory ST segment monitoring: Problems, pitfalls, solutions, and clinical application," *Brit. Heart J.*, vol. 44, pp. 419-425, 1980.

[3] V. Balasubramanian, "Validation of recorders and techniques," in *Proc. 3rd Int. Symp. Ambulatory Monitoring*, Harrow, England, 1979. London, England: Academic, 1980, pp. 504-506.

[4] J. W. Wagner, "A simple test for phase characteristics of chart recorders," *IEEE Trans. Biomed. Eng.*, vol. BME-26, pp. 505-508, Sept. 1979.

[5] Comm. Electrocardiogr., Amer. Heart Ass., "Recommendations for standardization of leads and of specifications for instruments in electrocardiology and vectorcardiology," *Circulation*, vol. 52, pp. 11-31, 1975.

[6] R. L. Longini, J. P. Giolma, C. Wall, and R. F. Quick, "Filtering without phase shift," *IEEE Trans. Biomed. Eng.*, vol. BME-22, pp. 432-433, 1975.

[7] D. P. Golden, R. A. Wolthuis, and G. W. Hoffler, "A spectral analysis of the normal resting electrocardiogram," *IEEE Trans. Biomed. Eng.*, vol. BME-20, pp. 366-372, 1973.

[8] P. H. Langner and D. B. Geselowitz, "Characteristics of the frequency spectrum in the normal electrocardiogram and in subjects following myocardial infarction," *Circ. Res.*, vol. VIII, pp. 577-584, May 1960.

[9] R. Cherry, C. Sanctuary, and H. L. Kennedy, "The question of frequency response," *Ambulatory Electrocardiol.*, vol. 1, pp. 13-14, June 1977.

[10] J. L. Myer, "Some instrument-induced errors in the electrocardiogram," *J. Amer. Med. Ass.*, vol. 201, pp. 351-356, 1967.

[11] A. S. Berson and H. V. Pipberger, "The low-frequency response of electrocardiographs, a frequent source of recording errors," *Amer. Heart J.*, pp. 779-789, June 1966.

Prosthesis Control Using a Nearest Neighbor Electromyographic Pattern Classifier

DAVID C. DENING, F. GAIL GRAY, AND ROBERT M. HARALICK

Abstract—**An investigation was conducted into the feasibility of applying a nearest neighbor algorithm to the problem of recognizing electromyographic (EMG) signal patterns for prosthesis control. A nearest neighbor algorithm correctly identified arm motions as belonging to one of six pattern classes from 72 to 100 percent of the time. A condensed nearest neighbor classifier was constructed to determine what minimum number of vectors was necessary in the look-up table.**

INTRODUCTION

The application of a nearest neighbor classifier to the problem of EMG pattern discrimination for prosthesis control was investigated. The amplitude of the EMG signals, detected by active electrodes from multiple sites, was input to a microcomputer which could serve as a data collection system or which could run a real-time nearest neighbor pattern classifier program. The microcomputer-based pattern classifier provided experience in the on-line training capability of a nearest neighbor classifier and an example of the classification accuracy that might be expected.

The differential EMG signal induced on the electrode plates was amplified and low-pass filtered to remove noise above the 1500 Hz frequency content of the desired signal. The signal was then high-pass filtered above 100 Hz to reduce 60 Hz pickup and to eliminate the very low frequency (less than 10 Hz) noise created by slight movements of the electrodes (motion artifacts). After half-wave rectification of the signal to obtain its amplitude envelope, the envelope signal was low-pass filtered for smoothing. This final output signal from the active

Manuscript received July 7, 1982; revised January 27, 1983.

D. C. Dening is with the Electronics Laboratory, General Electric Company, Syracuse, NY 13221.

F. G. Gray and R. M. Haralick are with the Department of Electrical Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061.

electrodes was a voltage proportional to the muscle contraction levels under the pickup electrodes.

The set of voltages obtained from the electrode complement was treated as a data vector. Four electrode pairs were employed in this evaluation; therefore, data vectors and the total data space contain four dimensions. The ends of the data vectors were found to cluster when plotted in the data space. Each cluster was associated with a set of vectors obtained from a distinct limb position and it was also found that the clusters tended to elongate radially outward from the origin. Since a nearest neighbor pattern (cluster) recognition algorithm was to be implemented, it was felt that the accuracy could be improved and the size of the look-up table minimized by normalizing the data vectors and using direction cosines instead of the voltages as they were input into the microcomputer.

Performance of the Micromputer-Based Classifier

Four electrodes were placed around a normal subject's right midforearm in roughly the four quadrants. Then five vectors were recorded for each of six limb configurations and entered into the look-up table. The limb configurations used were the following: hand grasp, hand open, wrist flex, wrist extend, forearm pronate, and forearm supinate. During the experiment, the subject received immediate feedback of any errors, and as a result an improvement in accuracy was obtained that was attributed to a learning process. The experiment was conducted by proceeding through the list of limb orientations, both forward and backward, and by requesting a position and then activating a single vector classification in the microcomputer.

The confusion matrix resulting from this total run of 600 vectors is shown in Table I. The vector labels in the left column indicate the pattern class for which a vector was requested. The numbers across each row are the percentage of time each class was chosen by the microcomputer classifier.

TABLE I
Confusion Matrix for the Total Test

INPUT VECTOR CLASS:	VECTOR CLASSIFIED AS: GRASP	OPEN	FLEX	EXTEND	PRON	SUPIN
GRASP	99	0	0	0	0	1
OPEN	0	83	0	0	17	0
FLEX	0	0	100	0	0	0
EXTEND	3	0	0	97	0	0
PRONATE	0	8	14	4	63	11
SUPINATE	3	1	0	0	18	78

TABLE II
Confusion Matrix for the Last Half of the Test

INPUT VECTOR CLASS:	VECTOR CLASSIFIED AS: GRASP	OPEN	FLEX	EXTEND	PRON	SUPIN
GRASP	98	0	0	0	0	2
OPEN	0	84	0	0	16	0
FLEX	0	0	100	0	0	0
EXTEND	0	0	0	100	0	0
PRONATE	0	12	6	6	72	4
SUPINATE	4	0	0	0	10	86

These results compare favorably to those reported by Herberts *et al.* [1] who used a linear combination discriminant function as a classifier to the EMG signal amplitudes from six sites and positioned the electrodes for optimum pattern separation. In both cases, the same limb configurations were used. Before the training, Herberts obtained correct classification accuracies ranging from 57 to 100 percent, and the majority of the classes were identified correctly more than 95 percent of the time.

The effect of the limited training that occurred during the course of the experiment can be seen by comparing the confusion matrix obtained for the last half of the experiment, shown in Table II, with that obtained for the entire experiment. As is evident in a comparison of the two tables, the classification of the first four movements (grasp, open, flex, and extend) did not improve substantially. However, the accuracy with which the last two classes were identified, i.e., pronate and supinate, did improve through the training process.

Further enlightenment as to why these particular results were obtained may be found from an examination of the look-up table used by the microcomputer. This is shown in Figs. 1 and 2 for two three-dimensional projections of the four-dimensional data space. The vector codes are A-grasp, B-open, C-flex, D-extend, E-pronate, and F-supinate. As may be seen best in Fig. 2, the vectors for classes A, B, C, and D formed reasonably tight groupings. Note that the operation of this classifier will place the unknown vector into the look-up table space (as is shown in the figures) and will pick the class of the closest vector. There was one "flier" from class A (grasp) that is close to class D (extend) and class F (supinate) and is probably responsible for those 3 percent errors shown in Table I. Class E (pronate) and F (supinate) were the most diffuse clusters, an indication that the unknown input vectors from these classes also had a corresponding spread. In addition, these diffuse clusters can sabotage the other classification results. Cluster B, corresponding to hand open, contains two reasonably close vectors from cluster E (pronate). As a result, it is not surprising that a vector generated by a hand open movement could fall closer to one of the intruding class E vectors than one from the correct class B cluster. This is reflected in the confusion matrix as the 17 percent erroneous classification of forearm pronate for vectors generated by a hand open motion.

The results reported in this section show what may typically be expected from this application of a nearest neighbor pattern classifier to the myoelectric prosthesis control problem. There are obvious places for improvement. For example, by judicious positioning of the electrodes, the discrimination of vectors for the pattern classes could be improved, as was indicated by Herberts *et al.* In addition, a fine tuning of the look-up table could eliminate many of the errors caused by the intrusion of "bad" vectors from one class into the domain of another class.

Condensed Nearest Neighbor Algorithm Evaluation

The previously described experiment using the microcomputer-based classifier provided an indication of the error rate with five vectors for each class template. An attempt was then made to determine the necessary size of the look-up table and to determine the resulting classification accuracy.

Two complete data sets of the EMG vectors were recorded for the limb configurations: hand grasp, hand open, wrist flex, wrist extend, forearm pronate, and forearm supinate. Both test data files, containing equal numbers of vectors (100) for each of the six pattern classes, could be accessed by the pattern classifier programs for training and evaluation. This method of algorithm evaluation removed any possibility of subject learning, which had been observed in the microcomputer evaluations. The resulting classification scores were thus lower than those reported earlier.

A condensed nearest neighbor classifier [2] was constructed to determine a typical size for the reference list template from which all unknown vectors are measured, and to compare its performance with that of the parametric classifier. The construction procedure for a condensed nearest neighbor classifier proceeds as follows. The reference list is seeded with one of the unknown vectors in the training set. Then all of the data vectors, in turn, are classified by measuring the distance (geometrically in this case) from the unknown vector to all the vectors in the reference list. The class of the nearest vector in the reference list is assigned to the unknown. If the classification is incorrect, that data vector is then added to the reference list. The procedure of classifying all of the training set is re-

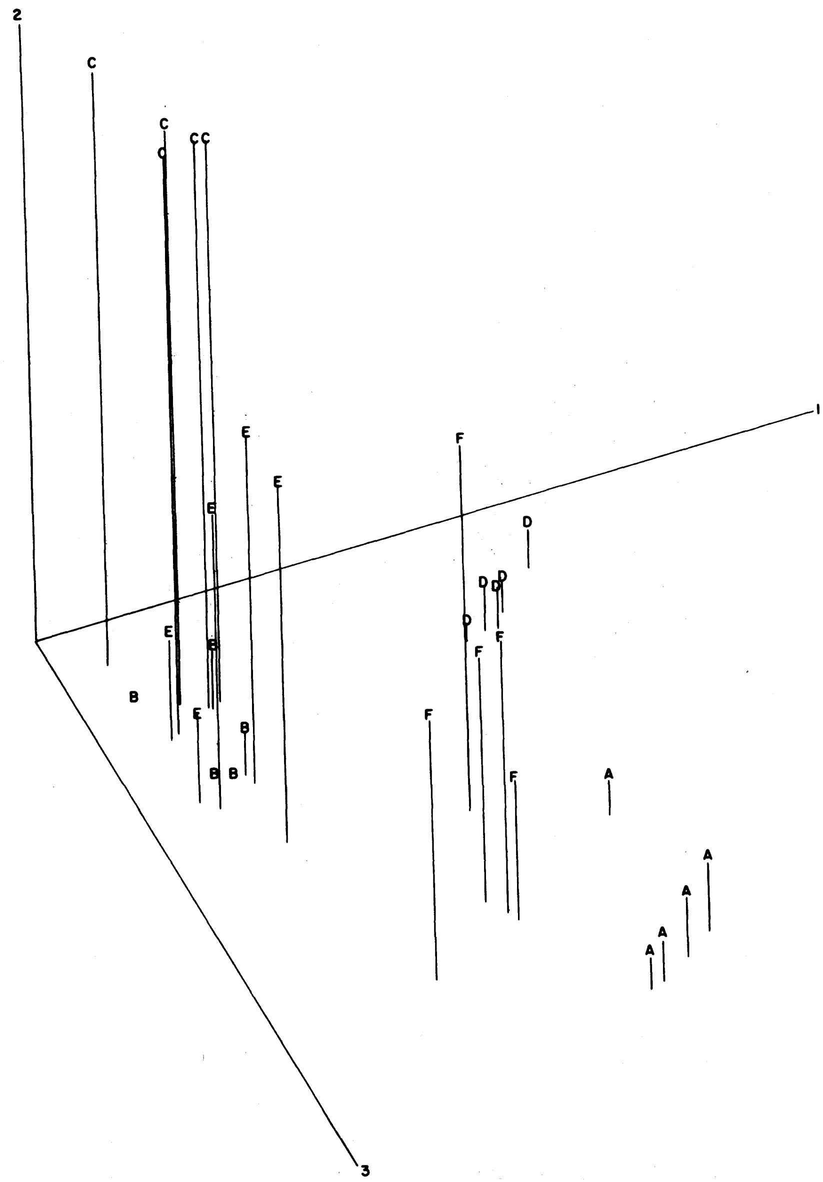

Fig. 1. Microcomputer look-up table projected onto the X1, X2, X3 dimension.

peated until either all data vectors can be classified correctly, or all the training set has been transferred to the reference list. However, in all trials this procedure converged with less than 23 percent of the training set in the reference list.

As is illustrated in the literature [3], this type of classifier tends to pick the initial reference list vectors randomly in the data space, but becomes more selective later as the decision boundary becomes better defined. The vectors chosen later tend to lie close to the decision boundaries. A further processing procedure was employed to prune the unneeded vectors from the reference list. In turn, each vector in the reference list was removed and the resulting list was used to classify the entire training set. If the entire training set was not correctly classified, the removed vector was placed back in the reference list. This pruning process reduced the reference list by 11–14 percent.

After the reference list was pruned, it was used as a template to classify the other data file (unknown vectors). The procedure was then repeated. The second data file was used to create the condensed reference list and the first data file was used for classifier evaluation. The combined results of training on one set and classifying the other for both data files is shown in the confusion matrix in Table III.

It was found that the number of vectors in the reference list tended to be low for the cases in which the classification accuracy was good. It was observed from studying the data files that the vectors for the classes "hand open" and "forearm supinate" tended to overlap in the data space and, as a result, were difficult for the algorithm to classify. This finding was also reflected in the relative class populations in the reference list for the various classes. The reference lists constructed in classifying the two data files contained the following average numbers of vectors per class.

Grasp	6
Open	39.5
Flex	1.5
Extend	6

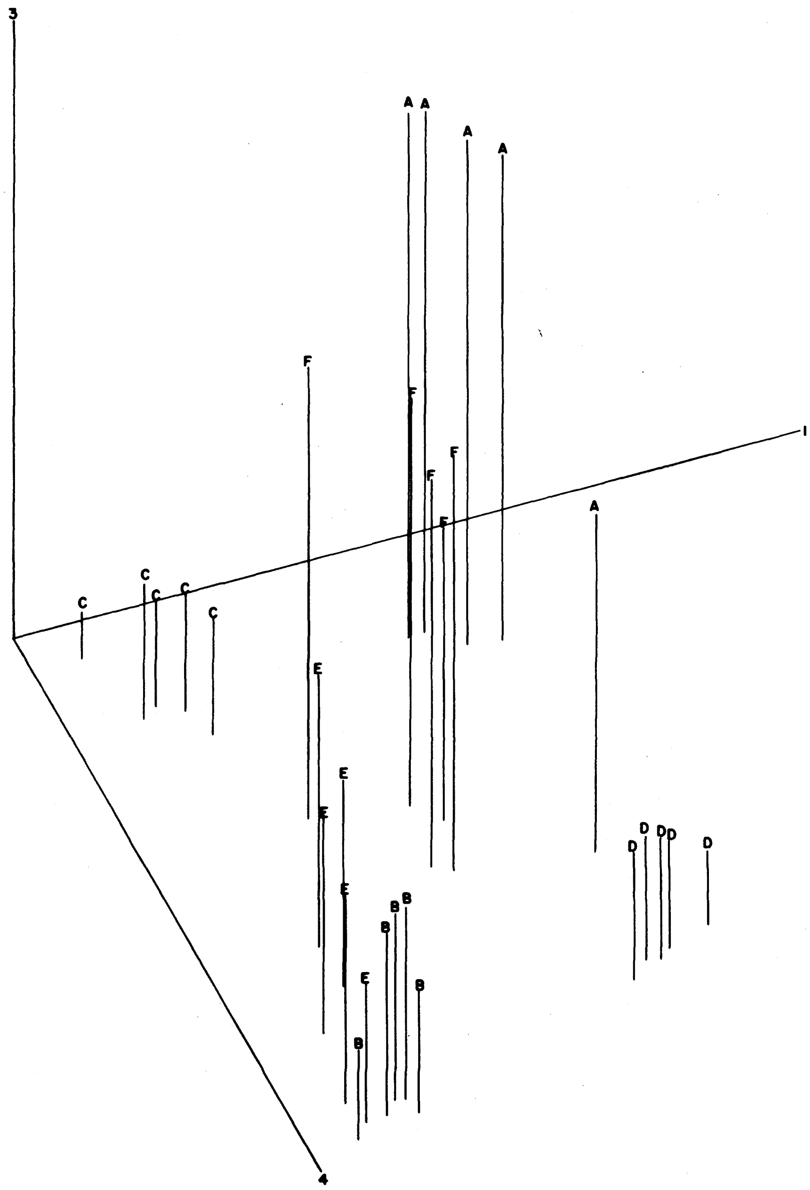

Fig.2. Microcomputer look-up table projected onto the X1, X3, X4 dimension.

TABLE III
CONFUSION MATRIX RESULTING FROM A NEAREST NEIGHBOR CLASSIFIER USING A CONDENSED REFERENCE LIST

INPUT VECTOR CLASS:	VECTOR CLASSIFIED AS: GRASP	OPEN	FLEX	EXTEND	PRON	SUPIN
GRASP	91	4.5	0	3	0	1.5
OPEN	.5	39.5	0	0	0	60
FLEX	0	0	99.5	0	0	.5
EXTEND	9	0	0	91	0	0
PRONATE	1	6.5	0	2.5	89	1
SUPINATE	.5	34.5	2	0	0	63

Pronate	5
Supinate	46.

This distribution provides a useful check on the reference list size needed for class separation. For easily distinguished clusters such as grasp, flex, extend, and pronate, only a few vectors are required to produce good classification accuracies.

DISCUSSIONS AND CONCLUSIONS

When the microcomputer-based nearest neighbor classifier was evaluated, five vectors were placed into the reference list. This decision was based on previous qualitative evaluations of the classifier and a desire to keep the cycle time of the classifier as low as possible. The validity of this decision was confirmed during the algorithm evaluations when the condensed reference lists were found to average between two and six vectors for most classes. The exceptions were the difficult to separate hand open and forearm supinate motions, which were heavily represented in the reference list because of the criteria used to classify a vector correctly during training or to add that vector to the reference list.

An inconsistency was observed between the results reported for the microcomputer-based pattern classifier and the results obtained from the prerecorded data files. As was reported for the microcomputer-based system, the patterns for pronate and supinate were difficult to resolve with the same accuracy as the other patterns. When the classifier was applied to the prerecorded data files, the difficult patterns to classify were open and supinate. The reason for this pattern difference is probably related to electrode placement and the fact that the two experiments were performed with different individuals.

In this experiment, the four electrodes were positioned around the four quadrants of the midforearm. A minimal amount of electrode repositioning was employed to optimize the correct pattern classification. In both cases, when the vectors were classified in the microcomputer and when data were recorded for the data files, the complete data sets were processed in one session to avoid the possibility of errors in replacing the electrodes.

Subject training was observed during the microcomputer-based pattern classification. In addition, an observation was made that various other EMG patterns could be generated by motions different from those of the six classes employed. With additional training, the nearest neighbor classifier could be used to map unorthodox but easily distinguished limb configurations for the desired classes.

References

[1] P. Herberts, C. Almstrom, and K. Caine, "Clinical application study of multifunctional prosthetic hands," *J. Bone and Joint Surgery*, vol. 60-B, pp. 552–560, Nov. 1978.

[2] P. E. Hart, "The condensed nearest neighbor rule," *IEEE Trans. Inform. Theory*, vol. IT-14, pp. 515–516, May 1968.

[3] I. Tomek, "Two modifications of CNN," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-6, pp. 769–772, Nov. 1976.

Dispersion of the Somatosensory Evoked Potential (SEP) in Multiple Sclerosis

KIM B. ROBERTS, PETER D. LAWRENCE, AND ANDREW EISEN

***Abstract*—A fast Fourier transform technique was used to quantitate the dispersion of the somatosensory evoked potential elicited by median nerve stimulation. The evaluation was proportional to the log of the SEP spectral energy between 380 and 1000 Hz. This was linearly scaled to a range of numbers between zero and five representing minimal to severe degrees of dispersion, respectively. In 21 controls, dispersion measured 1.5 ± 0.6 with an upper normal limit of 2.5. In 24 MS suspects, dispersion ranged between 0.3 and 3.9. A Bayesian decision-maker, based only on spectral energy, correctly classified all of the control group and 63 percent of a patient group with definite, probable, or suspected MS. Dispersion of the SEP, when used along with other factors such as latency and shape, adds to overall diagnostic accuracy.**

Manuscript received March 22, 1982; revised January 27, 1983. This work was supported by the Multiple Sclerosis Society of Canada.

K. B. Roberts and P. D. Lawrence are with the Department of Electrical Engineering, University of British Columbia, Vancouver, B.C., Canada.

A. Eisen is with the Division of Neurology and the Department of Diagnostic Neurophysiology, University of British Columbia, Vancouver, B.C., Canada.

Introduction

Visual, auditory, or somatesthetic stimuli can be used to induce electrical events, recordable over the cerebral cortex as cerebral evoked potentials (EP's). These potentials are termed, respectively; visual, auditory, and somatosensory potentials. Somatosensory evoked potentials (SEP's) were first recorded by Dawson in 1947 [1] without the aid of computer averaging. However, two and one-half decades elapsed before their use was popularized, requiring the advent of relatively sophisticated electronic equipment. Evoked potential recording is presently routine in many clinical neurophysiological laboratories throughout the world.

Among the various diseases in which SEP's are useful, multiple sclerosis (MS) ranks high [2], [3]. This disease, which has an incidence of about 100 per hundred thousand in Canada, is presently of unknown etiology. It results in loss and destruction of the myelin covering of central nervous system axons. Consequently, conduction in these fibers is slowed and eventually fails [4]. The altered conduction distorts impulse traffic, which is interpreted at a conscious level as a variety of inappropriate sensations. Varying degrees of motor dysfunction occur for similar reasons. In clinically definite MS [5], evoked potentials are frequently abnormal, approaching 100 percent when the system tested (i.e., visual, brainstem auditory, or somatosensory) is overtly involved [2], [3]. However, in suspected MS, a stage of the disease in which there is the greatest need for confirmation, EP's have a diagnostic yield generally below 40 percent. This partially reflects the limited means that have thus far been used to evaluate the EP quantitatively.

Characteristics of the SEP which have been or are potentially measurable include its latency, amplitude, morphology (or overall shape), and dispersion (or smoothness). These in turn depend, respectively, upon the speed of impulse traffic, the number of cortical or subcortical units that are simultaneously activated at each generator site, the number of normal neural generators that are sequentially excited, and the synchrony with which impulse traffic travels. The first three of these give useful information. However, desynchronization with resulting increased dispersion of the SEP can be expected to occur earlier and to a greater extent than alteration of latency, morphology, and amplitude [6].

In this study, we describe a method for measuring dispersion of the SEP, a characteristic that has thus far not been quantitated. The results in a normal control group are compared to a group of patients with suspected or definite multiple sclerosis [5].

Methods

Subjects and Patients

There were 21 normal volunteers without a relevant history of neurological disease who gave informed consent for study. Their mean age was 35.5 years (range 21–63 years). Eleven were women. 7 patients with definite, 14 with probable, and 3 with suspected MS, classified according to MacDonald and Halliday [5], were similarly studied. Their mean age was 36.2 years (range 18–60), and 10 were women.

Electrophysiological Techniques

Previously reported methods were used [7], [8]. In essence, the second digit was stimulated percutaneously using a DISA model 15E07 stimulator with monophasic square-current pulses applied through ring electrodes and an intensity 2.5 times sensory threshold (usually between 5 and 10 mA). Stimulus rate was 5/s and stimulus duration was 0.2 ms. Recordings were made from the scalp using needle electrodes po-

0018-9294/83/0600-0360$01.00