The Discrimination of Winter Wheat Using a Growth-State Signature

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In this paper, we describe a multitemporal classification procedure for crops in Landsat scenes. The method involves the creation of crop signatures which characterize multispectral observations as functions of phenological growth states. In this approach, crop spectral reflectance is modeled explicitly as a function of maturity rather than a function of date. This means that instead of stacking spectral vectors of one observation on another, as is usually done for multitemporal data, we establish for each possible crop category a correspondence of time to growth state which minimizes the difference between the given multispectral multitemporal vector and the category mean vector indexed by growth state. The results of applying it to winter wheat show that the method is capable of discrimination with about the same degree of accuracy as more traditional multitemporal classifiers. It shows some potential to label degree of maturity of the crop without crop condition information in the training set.

0.0 Literature Review: The Use of Landsat Multitemporal Data in Automatic Vegetation Mapping

The use of Landsat multitemporal data for classification of vegetation is a recent technique made possible by the capability for doing accurate registration of data from two or more observations of the same area. Through the use of multitemporal data, it may be possible to produce very accurate vegetation maps and crop acreage estimates. The reasons this increase in information could increase classification accuracy are:

(1) Different sets of classes may be separable at different times of the year.
(2) The measurements between classes may be more easily separable in a higher dimensional measure space.

This effect is due to the correlation between measurements at different times for the same class.

Classification of water, shrubs, and trees in the Great Dismal Swamp using a winter Landsat scene and a spring Landsat scene corresponded well with a vegetation map made from aerial infrared photographs (Gammon and Carter, 1976). In the winter scene, coniferous vegetation and standing water were spectrally distinct and therefore separable, whereas in the spring scene, the classes of deciduous vegetation were separable.

Other investigators have experimented with classification of crop and forest land (Von Steen and Wigton, 1976; Megier, 1977) and have reported increases in classification accuracy using multitemporal Landsat data. Von Steen and Wigton reported an 88% increase in overall classification accuracy of cotton, corn, soybeans, and grass over the best single-date classification accuracy of 50.8% using three observation dates late in the growing season.

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Some investigators have noted that the benefits of multitemporal Landsat data are dependent upon data acquisition dates (Landgrebe, 1974; Kalensky and Scherk, 1975). Landgrebe reported a decrease in classification accuracy of corn and soybeans in Illinois using the best date/band combinations from three observations: August 9, September 12, and October 2. However, classification using spectral data from September 12 and October 2, late season, and therefore, non-optimal observation dates, was better than classification using data from either of these dates alone. Similarly, Kalensky and Scherk noted that classification accuracy of forest maps was not improved significantly by multitemporal classification, but was consistently close to the best observation date classification. It seems then that higher dimensionality of multitemporal information can be used to achieve good classification of vegetation when optimal observation dates are not available, such as when an agricultural scene is obscured by cloud cover on mid-season dates of Landsat overpasses.

Most investigators have used maximum likelihood supervised discrimination, with the assumption that the measurement vectors from each class of data have a Gaussian distribution. In the simplest case, the covariance for each vegetation class is assumed to be the same and the maximum likelihood rule assigns measurements to the class whose mean measurement is nearest. If the a priori probability of each class is assumed to be the same, the measure of “nearness” is the Mahalanobis distance,

\[(x - \mu_i) \sum^{-1} (x - \mu_i),\]

where
\[x = \text{measurement to be classified},\]
\[\mu_i = \text{mean of the } i\text{th category},\]
\[\sum = \text{the covariance matrix}.\]

Multitemporal data has been used in the application of nonparametric classification techniques as well. LeToan et al. (1977) have tested methods of supervised nonparametric discrimination to estimate acreages of rice fields. The best rice acreage estimate with multitemporal data used barycentric distance. The barycentric distance between two measurements \(x_1 = (x_{11}, \ldots, x_{1N})\) and \(x_2 = (x_{21}, \ldots, x_{2N})\) is a weighted Euclidean distance,

\[d^2(x_1, x_2) = \sum_{i=1}^{N} k_i (x_{1i} - x_{2i})^2,\]

where the weights \(k_i\) are chosen to optimize the classification of a sample within the agricultural scene. Kauth et al. (1977) has developed BLOB, an unsupervised clustering technique that incorporates both spatial coordinates and multitemporal multispectral Landsat gray tones into measurement vectors.

Methods of multitemporal discrimination using crop maturity have been developed for winter wheat. The usual shift in the composition of spectral data from agricultural fields as recorded by Landsat is the initial shift from brown to green as the crop emerges and the crop canopy obscures the underlying soil. The subsequent shift is from maximum greenness, just before the crop heads, to yellow and brown as the canopy senesces and is
harvested. This pattern can be observed on Landsat scenes if one has scenes corresponding to the relevant stages in the crop growth cycle. "Green" measures, transformations of the four Landsat MSS bands, have been developed that correlate well with the degree of greenness of a crop (Kanemasu, 1974; Engvall et al., 1977; Kauth and Thomas, 1976; Nalepka et al., 1977, Salmon-Drexler, 1977). Some classification methods have been developed which take advantage of the fact that winter wheat goes through the brown–green–brown cycle before other crops because it is planted in the fall (Misra and Wheeler, 1978; Kaneko, 1978; Engvall et al., 1977; Erickson and Nalepka, 1976). An interesting property of these classifiers is that they compare "greenness" at different observation times rather than using the absolute values of measures at the observation times. Engvall's "Delta Classifier" has successfully identified wheat proportion over large areas with very limited ground truth.

Other multispectral multitemporal work includes Carlson and Aspiazu (1975).

Multitemporal Landsat data is a valuable resource which is only beginning to be evaluated and utilized for vegetation mapping. Further research will certainly result in improved maps, better acreage estimates, and advances in the closely related area of yield estimation.

1.0 Phenological Discrimination Motivations

A fundamental problem in crop identification with Landsat data is the number of variables, in addition to crop type that influence observed spectral reflectance values from a crop canopy. Among these is the degree of crop maturity, or phenological stage, which can vary even within a small area at a given time. For example, Nalepka et al. (1977) has observed significant differences in the phenological stage of winter wheat between different fields in Kansas LACIE Intensive Test Sites and even between areas within the same field. Furthermore, it is possible for one field to be at the same stage of maturity as a neighboring field was 18 days earlier. Differences in growth stage are particularly significant in the later parts of the growing season of winter wheat due to the rapid changes in appearance that occur with maturation, cutting, and in some cases, tilling of the fields.

We have experimented with a crop discrimination method that takes account of and utilizes this growth-state factor. Multitemporal classification is usually carried out by simply appending the spectral reflectance vectors observed at one time with the spectral reflectance vectors observed at another time. Then one processes the new data set as if it were vectors, like a single observation data set. The usual crop signature is a set of these multitemporal and multispectral vectors associated with the crop type. We use a crop signature which consists of sets of multispectral vectors associated with crop-type growth states. Associated with each crop is an "Mth order signature" which is a set of \((M+1)\)-tuples \((g;\alpha_1,\ldots,\alpha_M)\) where \(g\) is a growth state for the crop and \((\alpha_1,\ldots,\alpha_M)\) is an ordered set of gray-tone values for a subset of size
$M$ of the four Landsat MSS bands. We say that a pixel is of a given crop if: (1) Each set of observed gray tones on a particular date is consistent with some growth stage $g$ described in the signature of that area, and (2) These $g$s are consistent with what we know about vegetation phenology: i.e., growth states at later dates must be more mature than growth states at earlier dates. Classification is done by eliminating categories which do not satisfy conditions (1) and (2). If more than one category is left after the process of elimination, then the pixel is unclassified.

To illustrate the meaning of this, consider a two-band simple first-order example. Suppose observations $(\alpha_1, \alpha_2)$ and $(\alpha'_1, \alpha'_2)$ of a small patch of ground are taken at times $t_1$ and $t_2$ using the bands $b_1, b_2$. This can be classified by determining, for each category $c$, the first growth stage $g_1$ such that $(g_1, \alpha_1)$ and $(g_1, \alpha_2)$ is in the signature for $c$. If there is no such growth state, then the category $c$ is not consistent with the observed spectral reflectance and $c$ is not a possible classification for the pixel. If there is not a later growth stage $g_2 > g_1$ of category $c$ such that $(g_2, \alpha'_1)$ and $(g_2, \alpha'_2)$ is in the signature for $c$, then $c$ is not a possible classification for the pixel. Also note that we may impose restrictions on the growth states because only certain growth states may be possible at a particular observation time. In that case, category $c$ will not be a possible choice, if the only growth states consistent with the observed spectral reflectance are not possible for the observation times.

The implementation of this discrimination method requires two basic steps: (i) signature creation using a training set and (ii) classification of the multitemporal image using the derived signatures and crop calendar information. We experimented with using field-average vectors and vectors of randomly chosen individual pixels within ground fields for training data. We have tested the method using first- and second-order signatures. The details of implementation in the first case is described in the following sections. Haralick et al. (1980) describes second- and higher-order signatures.

1.1 First-Order Phenological Discrimination

First-order category signatures can be derived from training sets with an iterative procedure consisting of a step of dynamic programming minimization followed by averaging. This procedure is very much in the spirit of the ISODATA clustering technique (Ball and Hall, 1965). Let us restrict our attention to one category for the moment. Let $x(i,j,t)$ be the observed spectral reflectance in the $i$th band, $j$th sample (pixel or average over a field) of one crop type, taken at the $t$th observation time. The set

$$\{x(i,j,t)|i = 1, \ldots, I; j = 1, \ldots, J; t = 1, \ldots, T\}$$

is the training set for this crop category.

A first-order category signature will be a function which gives for each band and growth state, the mean spectral reflectance for the category. Let $u$ be a category signature. Then, $u(g, i)$ is the mean $i$th band reflectance of a small-area ground patch of that category in the $g$th growth state. The iterative procedure begins with a spectral signature for the category and successively improves it.

We take for the initial mean signature the average of the training vectors whose
time components have been simply interpolated over time to describe intermediate growth states. For example, say we have 5 observations, 15 growth states, and \( \bar{\alpha}(1) \) and \( \bar{\alpha}(2) \) are the average reflections in the first band at the first and second observation times. Then

\[
\begin{align*}
1; & \bar{\alpha}(1), \quad 2; \bar{\alpha}(1) + \frac{1}{3} [\bar{\alpha}(2) - \bar{\alpha}(1)], \quad 3; \bar{\alpha} + \frac{2}{3}[\bar{\alpha}(2) - \bar{\alpha}(1)], \quad \text{and} \quad 4; \bar{\alpha}(2)
\end{align*}
\]

are in the initial signature \( u \) for the crop. On each iteration we find a monotonic mapping called \( m, (j, t) \to g \), which minimizes

\[
\sum_{t=1}^{T} \max_{i} |x(i, j, t) - u(m(j, t); i)|
\]

for every sample \( i \) using a dynamic programming procedure. Note that this allows samples at different observation times to map into the same growth state.

At the end of each iteration the mean signature is updated. Define a set \( A_g \) as the set of all sample observation time pairs which are mapped to growth state \( g \). The updated mean signature \( u' \) is defined as

\[
u'(g, i) = \sum_{(j, t) \in A_g} \frac{x(i, j, t)}{\# A_g}.
\]

The procedure iterates until it reaches a fixed point. Figure 1 shows the final mean signature created by this procedure.

The standard deviation by band and growth state are listed in Table 1 for the samples mapped into the 20 growth states shown in Fig. 1. The average standard deviation is 1.42 (by equally weighted average). This compares with the average sample standard deviation by band date of 2.88. Associating spectral reflectance with growth state instead of observation time has reduced the average standard deviation by half.

We then broaden the first-order signature. In the broadening process, \( (g, \alpha_i) \) is included in the signature if \( |\alpha_i - u(g, i)| < w \). We chose the ”signature width” \( w \) to be about twice the magnitude of the average standard deviation of pixel reflectance within the growth stage. Then for each band \( \alpha_i \) and growth state \( g \), there is an interval of length \( 2w \) centered on \( u(g, i) \) of gray-tone values in the signature, as shown in Fig. 1. We note that, given the degree of variation in sample standard deviation for the growth-state bands, a single width for all bands and growth states is probably not best, but is chosen for simplicity.

In the discrimination process, one chooses which bands in the signature to use. Observed gray-tone values for a pixel in these bands must fall within these intervals in order for the pixel to be identified as in growth stage \( g \). In the case where more than one growth-state identification is possible, the earliest growth state is selected. In order for a pixel to be identified as crop \( c \), each observation must be identified as being in a growth stage for crop \( c \) and the growth stages must be chronologically ordered, as mentioned before. One also has the option of using crop calendar information. This limits the growth stages to a specified range for each observation time.

1.2 Bayesian Perspective of Phenological Discrimination

In a Bayesian framework for phenological classification of vegetation, we initially assume that spectral reflectance is a function of vegetation category, growth stage, and calendar time. We ne-
TABLE 1  The averages (which constitute mean signature) and standard deviations by growth state and MSS band of subsamples of a 120 wheat pixel sample of the Morton County Intensive Test Site. The first row of numbers are band means and the second row of numbers are band standard deviations.

<table>
<thead>
<tr>
<th>Growth state 1 with 41 samples</th>
<th>Growth state 11 with 48 samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.07</td>
<td>9.71</td>
</tr>
<tr>
<td>1.97</td>
<td>1.12</td>
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<tr>
<td>21.66</td>
<td>10.90</td>
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<tr>
<td>11.88</td>
<td>1.10</td>
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<tr>
<td>20.41</td>
<td>0.85</td>
</tr>
<tr>
<td>1.74</td>
<td>1.27</td>
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<table>
<thead>
<tr>
<th>Growth state 2 with 31 samples</th>
<th>Growth state 12 with 24 samples</th>
</tr>
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<tbody>
<tr>
<td>15.68</td>
<td>12.04</td>
</tr>
<tr>
<td>1.53</td>
<td>1.34</td>
</tr>
<tr>
<td>18.45</td>
<td>13.33</td>
</tr>
<tr>
<td>11.65</td>
<td>1.46</td>
</tr>
<tr>
<td>22.48</td>
<td>1.24</td>
</tr>
<tr>
<td>1.34</td>
<td>1.33</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Growth state 3 with 33 samples</th>
<th>Growth state 13 with 21 samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.21</td>
<td>9.38</td>
</tr>
<tr>
<td>1.79</td>
<td>1.13</td>
</tr>
<tr>
<td>18.61</td>
<td>10.62</td>
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<tr>
<td>10.76</td>
<td>1.46</td>
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<tr>
<td>17.82</td>
<td>1.17</td>
</tr>
<tr>
<td>1.40</td>
<td>1.11</td>
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</table>

<table>
<thead>
<tr>
<th>Growth state 4 with 16 samples</th>
<th>Growth state 14 with 29 samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.75</td>
<td>11.41</td>
</tr>
<tr>
<td>2.30</td>
<td>1.25</td>
</tr>
<tr>
<td>16.50</td>
<td>12.83</td>
</tr>
<tr>
<td>9.00</td>
<td>1.56</td>
</tr>
<tr>
<td>10.37</td>
<td>1.16</td>
</tr>
<tr>
<td>2.64</td>
<td>1.63</td>
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</table>

<table>
<thead>
<tr>
<th>Growth state 5 with 52 samples</th>
<th>Growth state 15 with 21 samples</th>
</tr>
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<tbody>
<tr>
<td>6.85</td>
<td>17.39</td>
</tr>
<tr>
<td>0.86</td>
<td>1.93</td>
</tr>
<tr>
<td>8.02</td>
<td>17.24</td>
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<tr>
<td>9.37</td>
<td>1.77</td>
</tr>
<tr>
<td>12.46</td>
<td>2.07</td>
</tr>
<tr>
<td>0.99</td>
<td>1.41</td>
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<table>
<thead>
<tr>
<th>Growth state 6 with 37 samples</th>
<th>Growth state 16 with 46 samples</th>
</tr>
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<tbody>
<tr>
<td>4.49</td>
<td>14.83</td>
</tr>
<tr>
<td>1.39</td>
<td>1.77</td>
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<tr>
<td>4.43</td>
<td>15.46</td>
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<tr>
<td>7.95</td>
<td>1.47</td>
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<tr>
<td>12.54</td>
<td>1.57</td>
</tr>
<tr>
<td>0.95</td>
<td>1.99</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Growth state 7 with 19 samples</th>
<th>Growth state 17 with 18 samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.84</td>
<td>11.33</td>
</tr>
<tr>
<td>1.39</td>
<td>1.60</td>
</tr>
<tr>
<td>3.84</td>
<td>12.94</td>
</tr>
<tr>
<td>13.74</td>
<td>1.78</td>
</tr>
<tr>
<td>13.68</td>
<td>1.94</td>
</tr>
<tr>
<td>1.89</td>
<td>2.25</td>
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<table>
<thead>
<tr>
<th>Growth state 8 with 10 samples</th>
<th>Growth state 18 with 20 samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.90</td>
<td>14.50</td>
</tr>
<tr>
<td>1.51</td>
<td>1.12</td>
</tr>
<tr>
<td>7.20</td>
<td>15.40</td>
</tr>
<tr>
<td>11.90</td>
<td>1.07</td>
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<tr>
<td>17.50</td>
<td>1.06</td>
</tr>
<tr>
<td>1.63</td>
<td>0.85</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Growth state 9 with 31 samples</th>
<th>Growth state 19 with 26 samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.87</td>
<td>17.88</td>
</tr>
<tr>
<td>1.13</td>
<td>1.55</td>
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<tr>
<td>10.32</td>
<td>17.58</td>
</tr>
<tr>
<td>10.58</td>
<td>0.84</td>
</tr>
<tr>
<td>12.55</td>
<td>1.41</td>
</tr>
<tr>
<td>1.16</td>
<td>1.69</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Growth state 10 with 30 samples</th>
<th>Growth state 20 with 47 samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.77</td>
<td>21.02</td>
</tr>
<tr>
<td>0.99</td>
<td>1.67</td>
</tr>
<tr>
<td>9.10</td>
<td>19.70</td>
</tr>
<tr>
<td>15.87</td>
<td>1.61</td>
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<tr>
<td>14.70</td>
<td>1.51</td>
</tr>
<tr>
<td>1.29</td>
<td>2.14</td>
</tr>
</tbody>
</table>

Note: glect the effects of atmospheric haze, geomorphologic soil, and moisture variations, as well as climatic catastrophies.

Let $G$ be the set of possible growth stages. The growth stages depend on maturity, biomass, percent cover, and height of vegetation. We will assume that the growth stages of $G$ are ordered according to the natural maturing cycle which the vegetation undergoes. Let $\{t_1, \ldots, t_N\}$ be the set of observation times. The times in the set $T$ are naturally ordered by the relation earlier than or later than. Let $R$ be the set of possible reflectance values and $B = \{1, 2, \ldots, M\}$ be the set of $M$ wavelengths bands of spectral reflectance that can be observed by the sensor.

Let $x$ be a spectral reflectance vector of vegetation category $c$ in phenological growth stage $g$ at calendar time. We denote the probability of observing $(x, c, g)$ at a given calendar time $t$ by $P(x, c, g|t)$.

For multitemporal multispectral data, the probability function of spectral reflectance vectors $x_1, \ldots, x_N$ coming from a
FIGURE 1. Final mean wheat signature for Morton County test site with tolerance set.
small area ground patch of categories $c_1, \ldots, c_N$ in phenological growth stages $g_1, \ldots, g_N$ at calendar times $t_1, \ldots, t_N$, respectively, is denoted by

$$P(x_1, \ldots, x_N, c_1, \ldots, c_N, g_1, \ldots, g_N|t_1, \ldots, t_N).$$

To determine a Bayes rule, the probability $P(x_1, \ldots, x_N, c_1, \ldots, c_N|t_1, \ldots, t_N)$ must be computed. Now

$$P(x_1, \ldots, x_N, c_1, \ldots, c_N|t_1, \ldots, t_N) = \sum_{g_1} \cdots \sum_{g_N}$$

$$\times P(x_1, \ldots, x_N, c_1, \ldots, c_N, g_1, \ldots, g_N|t_1, \ldots, t_N)$$

$$= \sum_{g_1} \cdots \sum_{g_N}$$

$$\times P(x_1, \ldots, x_N|c_1, \ldots, c_N, g_1, \ldots, g_N, t_1, \ldots, t_N).$$

$$\times P(c_1, \ldots, c_N, g_1, \ldots, g_N|t_1, \ldots, t_N).$$

We assume that the reflectance $x$ depends only on crop type $c$ and growth stage $g$ so that the probability for observed categories and multispectral reflectances is

$$P(x_1, \ldots, x_N, c_1, \ldots, c_N|t_1, \ldots, t_N)$$

$$= \sum_{g_1} P(x_1|c_1, g_1)P(g_1|c_1, t_1)$$

$$\times \sum_{g_2} P(x_2|c_2, g_2)P(g_2|c_2, c_1, g_1, t_1, t_2)$$

$$\times \cdots \sum_{g_N} P(x_N|c_N, g_N)$$

$$\times P(g_N|c_{N-1}, c_N, g_{N-1}, t_{N-1}, t_N)$$

$$\times P(c_1, \ldots, c_N|t_1, \ldots, t_N) \quad (*)$$

In theory, the formula just derived could be used to determine a Bayes rule in the usual way. In practice, there are too many calculations to do to calculate the required probabilities. However, because the required probability has the form of a product, if any probability in the product is zero, then the product must be zero. And a Bayes rule would never make an assignment to a category with a zero probability. This fact can be utilized to make an efficient table look-up rule which uses vegetation phenology just by storing in the table(s) those regions in measurement space having non-zero probability.

The astute reader will undoubtedly wonder why such a decision scheme has any chance of working at all. Why can't it be that any spectral observation vector is possible for many growth states for most categories? The reason that this is not possible is empirical. The probability distributions are conditioned by crop growth state, so that the resulting conditioned probability distributions are expected to have much smaller variances than the usual unconditioned ones. The conditioned ones, are therefore, much more peaked.

2.0 A Mathematical Description of Classification Using Phenological Vegetation Signatures and Prior Constraints

In the previous section, we derived a formula for the probability of a small ground patch having corresponding vegetation types $c_1, \ldots, c_N$ with respective spectral reflectance vectors $x_1, \ldots, x_N$ at observation times $t_1, \ldots, t_N$. In this section we will show how this kind of representation for the probability can be used to define vegetation signatures and a classification method which can then be used to recognize vegetation type and
growth state in a structural pattern recognition manner which is implementable as a table look-up rule.

For simplicity of discussion, we will assume that for the observation times $t_1, \ldots, t_N$, the small ground patch being observed does not change vegetation type and that the vegetation itself matures in a normal manner. We will also allow for the possible use of prior information which would indicate that at given observation times only certain growth stages for the vegetation category are reasonable ones. Such prior constraints can come from historical crop calendar information, perhaps combined with a vegetation growth model that uses local weather, temperature, and moisture information.

Before we define prior constraints for a vegetation signature, we need to review our notational conventions. Let $G$ be the set of possible growth states for the vegetation category. The growth states will depend on the maturity, biomass, percent cover, and height of the vegetation. We will assume that the growth states of $G$ are ordered according to the natural maturing cycle which the vegetation undergoes. Let $T$ be the set of observation times. The times in the set $T$ are naturally ordered by the relation earlier than or later than. Let $R$ be the set of possible reflectance values and $B = \{1, 2, \ldots, M\}$ be the set of $M$ wavelengths bands of spectral reflectance that can be observed by the sensor. Each spectral return vector $x$ is a member of the set $R^M$. The $m$th component of $x$ is the spectral return using the $m$th wavelength band of $B$

If the vegetation category does not change over the period of observation so that $c_i = c$ for $i = 1, 2, \ldots, N$, then the probability (*) derived in the last section is

$$P(x_1, \ldots, x_N, c|t_1, \ldots, t_N) = \prod_{n=1}^{N} \left[ \sum_{g_n} P(x_n|c, g_n) \cdot P(g_n|c, g_{n-1}, t_{n-1}, t_n) \right] \cdot P(c).$$

(\text{**)}

A necessary condition for a Bayes rule to assign the multitemporal, multispectral vectors $x_1, \ldots, x_N$ to category $c$ is for $P(x_1, \ldots, x_N, c|t_1, \ldots, t_N)$ to be nonzero. Since this joint probability is a product, if the joint probability is nonzero, then every term of the product must be greater than zero. This means that for each $n$,

$$\sum_{g_n} P(x_n|c, g_n)P(g_n|c, g_{n-1}, t_{n-1}, t_n) > 0.$$  

The product in each term of the above sum is nonzero if and only if $P(x_n|c, g_n)$ and $P(g_n|c, g_{n-1}, t_{n-1}, t_n)$ are both nonzero. The term $P(x_n|c, g_n)$ is nonzero if and only if the spectral reflectance vector $x_n$ is possible for vegetation category $c$ in growth state $g_n$. The term $P(g_n|c, g_{n-1}, t_{n-1}, t_n)$ is nonzero if and only if the cultivation practices of the region being observed allow vegetation of category $c$ to be in growth state $g_n$ at time $t_n$ and the rate of growth of vegetation type $c$ is such that growth $g_n$ can be reached from state $g_{n-1}$ in the time period from $t_{n-1}$ to $t_n$.

The set $S$ is called a signature for a category if it contains those data vectors whose components are spectral reflectances having nonzero probability for a growth state of a category. Let $M$ specify number of observed band wavelengths.
Let $x = (\alpha_1, \ldots, \alpha_M) \in R^M$ be a full set of $M$ observed reflectances and let $B'$ be a subset of the possible bands in $B$. The first-order signature is defined with respect to only those bands in $B'$. The first-order signature $S_c$ of a category $c$, $S_c \subseteq G \times R \times B$, consists of all 3-tuples of growth state, reflectance, and band whose conditional probability is greater than zero,

$$S_c = \{(\theta, r, b) | G \times R \times B | \times \text{for some } b \in B', \|P|g, c| > 0\},$$

where the pair $(r, b)$ denotes reflectance value and corresponding band wavelength so that $r = \alpha_b$.

In an analogous way, a first-order observation relation $\theta$, $\theta \subseteq T \times R \times B$, consists of all those 3-tuples of observation time, reflectance, and band which have been measured.

$$\theta = \{(t, r, b) | T \times R \times B | \text{for some } b \in B' \text{ and for some } n, t = t_n \text{ and observed reflectance was } r \text{ on band } b\}. $$

At each observation time, one reflectance value is measured for each band. (There may be none if there is missing data.)

The set $C$ of prior constraints relating growth states to observation times and growth states at earlier times is

$$C = \{(t_1, g_1, t_0, g_0) | (T \times G)^2 | \text{for some } n, t = t_n \text{ and observed reflectance was } r \text{ on band } b\}. $$

To determine if an observation $\theta$ requires us to reject a vegetation category [that is, determine if (**) is nonzero], we will determine if for every $n$, (1) there exists a growth state $g_n$ such that $P(x_n | c, g_n)$ is nonzero, and (2) if the $g_n$ is such that $P(g_n | c, g_{n-1}, t_{n-1}, t_n)$ is nonzero. To do so, we will need to associate observation times to growth states.

Let $H$ be a subset of $T \times G$ so that $H$ associates observation times with growth states. We define the relation composition of $\theta$ with $H$, written $\theta \cdot H$ by

$$\theta \cdot H = \{(g, r, b) \in G \times R \times B | \times \text{for some } t \in T \times (t, g) \in H\}. $$

If $\theta \cdot H$ is not a subset of $S$, then (1) is not satisfied. If $\theta \cdot H$ is a subset of $S$, and $K = M$, and $H$ is defined everywhere on $T$, then (1) is satisfied. If $H \times H$ is a subset of $C$, and $H$ is defined everywhere on $T$, then (2) is satisfied.

Given $\theta$ and $S$, let us find $H$ so that $\theta \cdot H$ is a subset of $S$, and $H \times H$ is a subset of $C$, in the case where

$$C' = \{(g, t) | P(g | t) > 0\}. $$

That is, growth states are constrained to be chronologically ordered in time and consistent with observation times.

The table look-up implementation for finding $H$ is as follows. For each band $b$ in $B'$ there is a table $T(b)$, which gives lists of possible growth states for values of reflectances in band $b$. These tables together comprise the first-order signature $S$. There is a table $C'$ listing possible growth states for given observation times. The implementation works as follows.
Given \( \theta \subseteq T \times R \times B \), the existence of a function satisfying (1) and (2) is an easy matter to ascertain. Define \( F(b) \) by

\[
F(b) = \{(t, g) \in C' | \text{for some } r \in R, \times (t, r, b) \in \theta \text{ and } (g, r, b) \in S\}.
\]

Then with the properties of \( \theta \) already mentioned, it can be proved that any \( H \subseteq C' \) satisfying \( \theta \cdot H \subseteq S \) must be contained in \( \bigcap_{b \in B} F(b) \) and furthermore, the composition \( \theta' \cdot \bigcap_{b \in B} F(b) \) must be contained in \( S \).

This implies that we can determine the existence of a function satisfying (1) and (2) by construction. First construct the relation \( W \) in \( C' \) defined by

\[
W = \bigcap_{b \in B'} F(b).
\]

Then construct a monotonic part \( H \) of \( W \). If this \( H \) associates a growth state for each observation time, then \( H \) is a function satisfying (1) and (2).

Suppose \( t_1 \) is the first observation time. Using the table \( C' \) we retrieve a set of possible growth states \( G_1 \) and we check growth states in \( G_1 \) against observed reflectances until we find the earliest growth state consistent with the observed reflectances. We check a growth state in \( G_1 \) as follows: For each band \( b \) in \( B' \) we enter the corresponding observed reflectances at \( t_1 \) into the table \( T(b) \) and get back a set of growth states. If each such set contains the growth state we are considering, the growth state is consistent with the observed reflectances. At time \( t_2 \), we retrieve a set of possible growth states and intersect with the set of possible growth states later than the earliest consistent growth state for \( t_1 \) to get \( G_2 \). Then find the earliest growth state in \( G_2 \) which is consistent with the observed reflectances at time \( t_2, \ldots, \ldots \), and so on for each observation time.

### 2.1 Table Look-Up Rule Implementation (First-Order and No Prior Constraints)

In this section, we specialize the implementation discussed in Sec. 2.0 for the first-order signature case with no prior constraints on growth states. A sufficient condition for

\[
\sum_{g_n} p(x_n | c_n, g_n) p_{t_n}(g_n | c_n)
\]

to be zero is for \( p(x_n | c_n, g_n) = 0 \) for all values of \( g_n \). Let \( x \) be a \( K \)-dimensional spectral reflectance observation. A sufficient condition for \( p(x | c, g) = 0 \) for all growth values of \( g \) is for there to be no phenological growth stage \( g \) which gives a positive marginal conditional probability for each component of the observed reflectance \( x \). Let \( P_{1,\ldots,K}(\alpha_1,\ldots,\alpha_K | c, g) \) be the probability of observing the \( K \) spectral band reflectance \( (\alpha_1,\ldots,\alpha_K) \) from a vegetation of type \( c \) in growth state \( g \). Let \( P_k(\alpha_k | c, g) \) be the marginal probability of observing spectral reflectance \( \alpha_K \) from band \( K \) given vegetation type \( c \) and growth state \( g \). Then a sufficient condition for \( P_{1,\ldots,K}(\alpha_1,\ldots,\alpha_K | c, g) = 0 \) is for \( P_k(\alpha_k | c, g) = 0 \) for some spectral band \( k \). If there is no phenological growth state which gives a positive marginal conditional probability for each component of the observed spectral reflectance \( (\alpha_1,\ldots,\alpha_K) \), then

\[
\bigcap_{k=1}^{K} \{ g | P_k(\alpha_k | c, g) > 0 \} = \emptyset.
\]

This leads to the following criteria for eliminating category assignments which a Bayes rule would also eliminate.
For a given $\epsilon > 0$, define the table by $R(k, \alpha, c) = \{ g \mid P_k(\alpha|c, g) > \epsilon \}$. Suppose multitemporal multispectral returns of $(\alpha_{11}, \ldots, \alpha_{1K}), (\alpha_{21}, \ldots, \alpha_{2K}), \ldots,$

$(\alpha_{N1}, \ldots, \alpha_{NK})$

are observed for calendar times $t_1, \ldots, t_N$. Then if

$$\bigcap_{k=1}^{K} R(k, \alpha_{nk}, c) = \emptyset$$

for some $n$, a Bayes rule could not make the assignment to category $c$. If

$$\bigcap_{k=1}^{K} R(k, \alpha_{nk}, c^*) \neq \emptyset$$

for all $n$, and for every $c \neq c^*$,

$$\bigcap_{k=1}^{K} R(k, \alpha_{nk}, c) = \emptyset$$

for some $n$, then a Bayes rule must make the assignment to the unique category $c^*$.

Table look-ups are the most efficient way to do classification of a large data volume. It is significant that this technique submits to such an implementation. Reviews of more standard table look-up techniques can be found in Haralick (1976).

2.2 Example

An example easily illustrates the first-order table look-up idea graphically. Figure 2 shows graphs for the tables $R(k, \alpha, c)$. A square blacked in for coordinates $(g, \alpha)$ means that for the corresponding spectral value $\alpha$, the phenological growth-stage $g$ belongs to the table $R$. Suppose that there are two spectral wavelengths, band 1 and band 2, two categories, and two times at which observations are taken. Let the spectral observation for time 1 be $(9, 10)$ and the spectral observation for time 2 be $(3, 6)$. Examining the tables for category 1, we have

$$R(1,9,1) = \{3, 5, 6, 7\},$$
$$R(2,10,1) = \{0, 1, 2, 3, 17, 18, 19\},$$
$$R(1,9,1) \cap R(2,10,1) = \{3\}.$$

This means that the only time observation $(9, 10)$ could occur from category 1 is during phenological growth-stage 3. Examining the tables for category 2, we have

$$R(1,9,2) = \{5, 6, 7, 13, 14\},$$
$$R(2,10,2) = \{0, 1, 7, 8, 18, 19\},$$
$$R(1,9,2) \cap R(2,10,7) = \{7\}.$$

This means that the only time the observation $(9, 10)$ could occur from category 2 is during phenological growth-stage 7. So after the first spectral observation, both categories are still possible.

Now consider the second observation $(3, 6)$. By the tables,

$$R(1,3,1) = \{13, 14\},$$
$$R(2,6,1) = \{6, 7, 8, 9, 13, 14\},$$
$$R(1,3,1) \cap R(2,6,1) = \{13, 14\}.$$

This means that spectral observation $(3, 6)$ is possible for category 1 only during phenological growth-stages 13 and 14.

By the tables,

$$R(1,3,2) = \{0, 1\},$$
$$R(2,6,2) = \{11, 12\},$$
$$R(1,3,2) \cap R(2,6,2) = \emptyset.$$
This means that there is no phenological growth stage for category 2 which yields the spectral observation (3, 6). The conclusion, therefore, is that the small ground patch having early spectral return of (9, 10) and later spectral return of (3, 6) must be an area of category 1 vegetation observed during its 3 and 13 or 14 phenological growth stages.

If instead of the intersection \( R(1, 3, 2) \cap R(2, 6, 2) = \emptyset \), we had \( R(1, 3, 2) \cap R(2, 6, 2) = \{4, 6\} \), category 2 would be eliminated because the spectral reflectance it has at a late calendar time match a possible spectral reflectance for category 2 only at early phenological growth-states 4 or 6. Later calendar times must correspond to later phenological growth states.

### 3.0 Identification of Wheat in Morton County Using Phenological Discrimination Methods

An extensive investigation of the use of phenological discrimination was carried out using the Morton County image. Our results indicate that the method is capa-
ble of discriminating wheat with about the same degree of accuracy as the traditional approaches. However, the method does work with missing data and provides additional information on crop maturity. In this section, we discuss our results as well as the effects of the signature width and band choices on the quality of classification. Also, the validity of our dynamic programming method for creation of first-order mean signatures is illustrated.

3.1 A Discussion of First-Order Phenological Results

Consider the two steps in the first-order discrimination procedure. In the first step the user chooses an input sample to train the signature and the number of growth states to be characterized in the signature. In the identification step, the user chooses the “signature width” and which MSS band/observation date combinations to use. The choice of signature width is critical, especially when one is identifying only one crop class. The larger the signature width, the larger the number of pixels that will be identified as in the crop class. The percent correct identification will increase with width but at the cost of increased false identification. In the second step, the identification step, the user also has the option of specifying a range of allowed growth states for each observation time. A good choice of these growth state restrictions effectively cuts down on the number of false classifications, without much reduction in the rate of correct classification.

Sample adequacy was investigated by comparing the discrimination results with no growth-state restrictions using a sample of 35 wheat field averages and several random samples of individual pixels. It seems that a sample of around 100 pixels (about 2.5% of the ground-truth wheat) is of adequate size as discrimination was not significantly better with a sample of twice that size or with the field average samples.

We have performed four identifications of wheat with signatures having 5, 10, 20, and 36 growth states, respectively. This is a range of one to seven growth states per observation time, since we have five observations of the Morton County test site. The general shape of the mean signatures with differing numbers of growth states is the same. Our best discrimination was with a 36-growth-state signature with a width of 3.25. Using this signature and all observation dates, the results were 83% correct identification of ground-truth wheat and 4% false identification. With a 5-growth-state signature and a width of 6.0, the corresponding figures were 79% and 13%. The improved discrimination shows the usefulness of modeling several growth states per observation time.

The number of MSS bands needed for accurate identification was investigated. Most of our testing of the first-order discrimination procedure has been done using MSS bands 4, 5, and 6. However, it has been found that MSS bands 4 and 5 are sufficient for good wheat identification. Adding MSS band 7 reduced correct classification significantly. Before testing, it was thought that perhaps MSS bands 5 and 7 would be very useful for phenological discrimination of wheat, because they have often been most useful in other agricultural classification techniques. However, the identification of wheat with MSS bands 5 and 7 using the first-order phenological method turned
out to be not as good as with MSS bands 4 and 5.

The possibility of accurate wheat identification with a single channel of information per observation time was investigated. Since the phenological method of discrimination is a growth-stage identification process, it seemed likely that a single measure, indicating greenness of the pixel at the observation times, would be sufficient for identification of the crop. The four MSS band values for each observation date were transformed into Kauth greenness (KG) (Kauth, 1976), a linear combination of the band values scaled to fit in the 0–31 integer value range.

\[
KG = 0.514(-0.290 \text{MSS 4} - 0.562 \text{MSS 5} + 0.600 \text{MSS 6} + 0.491 \text{MSS 7}) + 13.6.
\]

Wheat identification with this measure was not as good as identification with two or three MSS bands.

Good wheat identification depends on the proper choice of growth-state restrictions, especially if a subset of observation times is used. A description of a run using only two observation times will illustrate this. In this run, possible growth states were restricted to states 1–5 for observation time 1 and states 10–12 for observation time 2. The small number of growth states allowed for the second observation time, May 9, is important because winter wheat is distinguished from other crop types principally because it is green on the May 9 date. Growth states 10–12 in the signature had low gray-tone values in MSS band 5, which shows that they correspond to green states. Given the preceding growth-state restrictions, 81% of the ground-truth wheat was correctly identified with 5% of the non-wheat cells incorrectly identified.

The best choice of observation times was October 23 and May 9 for first-order discrimination of wheat. The best single observation time turned out to be May 9, as expected. The October 23 observation turned out to be the best addition to the May 9 observation. A third observation improved results significantly only when wheat was broken into two categories, quickly maturing wheat and slowly maturing wheat. The same 36-growth-state signature was used to identify both subcategories of wheat, but with two sets of growth state restrictions. This discrimination resulted in a total of 83% of the wheat being identified, with only 4% false classification.

### 3.2 Testing the Validity of Dynamic Programming in First-Order Mean Signature Generation

Recall that different observation times map into the same growth state in the construction of the first-order mean signature. In order to test whether it is good to allow observations from different times to be used in the construction of growth state, an alternate procedure was tested. Let us say we have \( G_0 \) as the number of growth states per observation time. In each iteration we define a mapping \( m: (j, t) \rightarrow G \) which minimizes

\[
\sum_{t=1}^{T} \max_i |x(i, j, t) - u(m(j, t); i)|
\]

for each sample \( j \) with the additional restriction that the pair \((j, t)\) must map into one of the growth states in the set \((t-1)G_0 + 1, (t-1)G_0 + 2, \ldots, C_0t\). Because these sets are not overlapping, the method for finding the mapping turns out to be a simple minimization.
A few phenological discrimination runs using five observation dates were made using mean signatures generated by simple minimization. Discrimination was not quite as good as with similar runs using dynamic programming. The average standard deviation by band and growth state for the samples mapped into 20 growth states was higher with this simple minimization. This demonstrates the validity of combining observations with different dates in characterizing a signature growth state.

3.3 An Experiment with Use of Two Signatures for Wheat

First-order discrimination with a fairly small signature width results in about half the wheat being identified with a very small amount of false identification, when appropriate growth state restrictions are used. It was thought that perhaps wheat is better characterized by two or three signatures with small widths. Our experimentation did not lead to improved classification, but provides insight into the properties of the growth states in the signature.

A sequential procedure was used. Areas of wheat which were poorly identified by phenological discrimination were examined. It seemed that there were two types of wheat not being identified. One type was wheat with reflectances generally higher than average for all MSS bands on all observations. The other type was wheat with generally lower than average reflectances, especially for MSS bands 4 and 5 on the May 9 observation date. In order to try to identify these problem areas of "high" and "low" wheat, signatures were created from samples of wheat not yet identified. A "high" signature was created from pixels in the sample whose quantized values in MSS bands 4 and 5 on the May 9 observation date was below a threshold of 6. A "low" signature was created from pixels whose values in MSS bands 4 and 5 on the May 9 observation date was above 8. High and low wheat were classified with these signatures. Areas identified as high and low wheat were quite distinct.

In an attempt to determine the identity of these areas, aerial photographs of Morton County were examined. It was noted that small low wheat areas within fields were often near field borders, and were probably weedy areas. Areas classified as high wheat were often found in field locations that appeared to be on high ground or were composed of light-colored poor soil.

We also investigated the high and low wheat by looking at field mean of Kauth greenness (KG) and Kauth soil brightness (KSB). Kauth soil brightness is a linear combination of the MSS band which we rescaled to fit in the 0–31 value range

$$KSB = 0.522(0.433 \text{ MSS 4} + 0.632 \text{ MSS 5} + 0.568 \text{ MSS 6} + 0.264 \text{ MSS 7})$$

Those fields that were classified primarily as high wheat were areas of high KSB and about as much KG as fields with predominantly low wheat, except on the May 9 date when they were "greener".

We investigated further by examining the samples for the high and low signature. We looked at a 36-growth-stage signature created from a random sample of ground-truth wheat and found which growth states each observation of the sample mapped to. Low samples are mapped into relatively earlier growth
states compared to the high reflectance samples, except for the October 23 observation.

The explanation which seems most consistent in explaining the high and low areas is that high areas are poor quality stands of wheat, which were adversely affected either by the dry weather in Morton County in 1974 or by poor soil. The low areas are vigorous stands of wheat, or areas with a lot of weeds. Vigorous stands of wheat mature more slowly than stands maturing in less than optimal conditions. The dryer fields will be the first to head, and therefore, look less green on May 9.

4.0 Conclusion

The phenological growth state procedure seems to be able to discriminate wheat about as well as some more standard procedures and label degree of maturity as well.

It has the advantage of a table look-up implementation as well as being able to handle missing observation data (e.g., missing due to clouds) and being able to naturally use crop calendar constraint information. The experiments reported here need to be repeated with other crops and natural vegetation to assess its efficacy and validity.

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Received 22 January 1979; revised 10 December 1979.