A HYBRID SHAPE DECOMPOSITION USING HYPERQUADRICS AND MATHEMATICAL MORPHOLOGY

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ABSTRACT

This paper discusses a hybrid shape decomposition method using hyperquadrics to describe a part, mathematical morphology to find an initial ellipse, and polygonal approximation to find additional terms of the hyperquadrics. We adopt a weighted least square fitting to find hyperquadrics using the Levenberg-Marquardt optimization method. The proposed method is illustrated in decomposing some 2D binary images such as a natural object and a man-made object. It can be easily extended to 3D range data. The experimental results show that our method is a promising approach in shape representation.

1. INTRODUCTION

The perception of shape can be conceptualized as a process in which the shape is segmented into simple components that can be easily recognizable. From such a point of view, shape decomposition techniques can be considered the most desirable, since it often provides structural descriptions which are close to human intuition.

Shape decomposition methods can be classified into two approaches. One is the decomposition of a shape's interior region and the other is the segmentation of its boundary. A region based approach uses convex shapes as primitives such as generalized cylinders[1], superquadrics[2], disks, and structuring elements in morphology[3]. In contrast, boundary based segmentation uses boundary features such as high curvature points[4].

In this paper, we discuss a hybrid method which produces a structural description of decomposed parts from contour data or range data using a hyperquadric model, morphology, and polygonal approximation. The hyperquadric model and morphology are region based approaches, whereas polygonal approximation is a contour based approach.

First, we extract a maximal inscribable shape(MIS) from the object using morphological operations in order to find an initial ellipse and select a set of fitting data from the boundary points. Such a set of points can be extracted among the visible boundary points from the set of points in MIS. Next, the MIS is expanded to a maximal inscribable convex hyperquadrics (MICH), which contains the MIS and is maximally convex within the given object. MICH is obtained by the weighted least square fitting. To find a set of additional terms for the hyperquadrics, we extract a few line segments that are in the polygonal approximation of boundary points and touch the initial ellipse. We can decompose the object into several MICH by applying the above procedure recursively. In other words, we try to find the second largest MICH among the remaining segmented regions which lie outside of the first MICH. The method is verified by an experiment with many kind of shapes including a natural object and a man-made object.

2. RECURSIVE DECOMPOSITION

In this section, we present a recursive decomposition method by hyperquadrics and mathematical morphology. In the following notation, the subscript denotes the $i^{th}$ level and the superscript denotes the $j^{th}$ part in the $i^{th}$ level. Let $X$ be the object and $X_0 = X$.

First, an initial MIS(Maximal Inscribable Shape) $M_i^j$ for part $X_i^j$ is extracted by opening:

$$M_i^j = (X_i^j \ominus n_i^j B) \oplus n_i^j B$$

where $n_i^j$ denotes the maximal radius of the structuring element(SE) in the $i^{th}$ part $X_i^j$ such that, $X_i^j \ominus n_i^j B \neq \emptyset$. $B$ is a digital circle SE of 5 by 5.
Next, \( M_i^j \) is expanded to MICH(Maximal Inscribable Convex Hyperquadrics) \( f(x) \) within the part \( X_i^j \) by a weighted least square fitting. Let \( H_i^j \) be the inner area of a MICH \( f(x) \):

\[
H_i^j = \{ x \mid f(x) \leq 0 \}
\]

For the next recursion step, \( H_i^j \) is removed from \( X_i^j \) and the resulting residue is opened with a small structuring element to remove small hair lines like pieces. The connected components \( X_i^{j+1}, \; (j = 0, \ldots, k_i - 1) \) in \( (X_i^j - H_i^j) \circ K_{\text{small}} \) constitute the parts of the shapes at the \((i+1)\)th level of the decomposition hierarchy.

This recursion step for part \( j \) at level \( i \) stops when \( n_i^j \) is zero. The final decomposition is a set of \( H_i^j \) \((j = 1, \ldots, k_i - 1; \; i = 0, \ldots, I - 1) \). The total number of the decomposed parts is \( \sum_{i=0}^{I-1} k_i \). This recursive decomposition process results in a hierarchical representation of parts. The largest part \( H_0^I \) is at the top of hierarchy. The smallest parts are in the bottom level. The original shape can be approximated by \( \hat{X} \approx \bigcup_i \bigcup_j H_i^j \).

In the section 4, we will explain how to extract \( H_i^j \).

3. HYPERQUADRATIC MODELS

Hyperquadrics are generalization of superquadrics and can describe more complex shapes with a compact form. Hanson[5] extended superquadrics and proposed hyperquadrics for computer graphics applications. Han[6] used hyperquadric model for shape recovery from range data. However, they have decomposed the shape manually because of the difficulty of decomposition. In this paper, we will use hyperquadric model for shape decomposition due to the following properties of hyperquadrics:

- **description power** : The hyperquadrics can describe any shapes including triangle, rectangle, circle, half circle, ellipse, and even a concave shape.
- **semi-local control** : By adjusting a term in hyperquadrics, we can change the shape in the direction specified by the term.
- **convexity and concavity** : The shape of hyperquadrics is convex, if all exponents are greater than one. If some terms has an exponent less than one, the result shape is concave.

The equation of hyperquadric model is defined as [5].

\[
\sum_{i=1}^{n_i} |a_i x + b_i y + c_i z + d_i|^{\gamma_i} = 1
\]

where \( n_i \) is the number of terms, that is, a pair of bounding planes, \( |a_i x + b_i y + c_i z + d_i| = 1 \), \((i = 1, \ldots, n_i)\), and \( \gamma_i \geq 1 \) for convex hyperquadrics.

Let \( x = (x, y, z) \), \( \alpha = (a_1, b_1, c_1, d_1, \gamma_1, \ldots, a_{n_i}, b_{n_i}, c_{n_i}, d_{n_i}, \gamma_{n_i}) \). Then,

\[
f(x; \alpha) = 1 - \sum_{i=1}^{n_i} |a_i x + b_i y + c_i z + d_i|^{\gamma_i}
\]

provides an “inside/outside” test that determines the relationship between a point and the surface. In the following sections, \( f(x) \) is restricted to 2 dimensions.

4. FITTING OF HYPERQUADRICS

We use a weighted least square fitting to find MICH rather than an equally weighted, because MICH is not the normal least square of boundary points but the boundary of the central space of the object that has maximal convex area within the object.

The initial guess is given as an ellipse that can be calculated from \( M_i^j \). The ellipse is fitted using \( \{X_i^j\} \), the boundary points of \( X_i^j \), by the same fitting algorithm on hyperquadrics of two terms, each with an exponent having value 2. An initial guess for the higher order terms can be extracted from the line segments which are in a polygonal approximation of \( \{X_i^j\} \) and touch the ellipse. For a robust estimate of the hyperquadric parameters, the data to be fitted from \( \{X_i^j\} \) is limited to the set of visible points from the points in \( M_i^j \).

4.1. Extraction of Visible Points

The set of visible points from the center of \( M_i^j \) may include points in a concave part of the shape, where the concavity is determined at the inner space of \( M_i^j \). The points in the concave part can be removed by testing visibility at the several points of \( M_i^j \). An attempt to test all points in \( M_i^j \) requires too much computation time. In the experiment, 8 test points at the 8 directions are sufficient. Such test points are determined as follows. Let \( \mu \) be a centroid of \( M_i^j \).

\[
d_m = \min_{m \in M_i^j} D(\mu, m)
\]

where \( D(\mu, m) \) is Euclidean distance. Let \( T_i^j \) be a set of test points such that

\[
T_i^j = \{ (x, y) | x = d_m \cos \theta, \; y = d_m \sin \theta, \theta = \frac{k \pi}{4} \; (k = 0, 1, \ldots, 7) \}
\]
Let \( V^j_t \) be a subset of \( (X^j_t) \) and all elements in \( V^j_t \) is visible from the points in \( T^j_t \). The visible points are easily extracted by checking the intersection between each line segments which are in the polygonal approximation of shape boundary \( (X^j_t)_t \) and the line segment formed where one point is from \( (X^j_t) \) and the other is from \( T^j_t \). Let \((a, b)\) be a line segment which has two end points \( a \) and \( b \), and the polygonal approximation be the sequence of points, \((p_0, \ldots, p_{n_t-1})\). Then, \( V^j_t \) is defined by

\[
V^j_t = \{ x \in (X^j_t)_t | \text{for every } t \in T^j_t \}
\]

and for \( l = 0, \ldots, n_t - 1, \)

\[\text{Intersect}((p_l, p_{l+1} \mod n_t), (x, t))\].

Figure 1.(b) shows \( V^j_t \) for the wings of F18 aircraft.

4.2. Weighted Least Square Fitting

It would be difficult to determine a robust estimate of the hyperquadric parameters, if the set of points to be fitted will change severely at each iteration step due to the thresholding. Hence, the set of data to be fitted is limited to \( V^j_t \) at the most. Let \( x_k \in V^j_t = (x_1, \ldots, x_{N^j_t}) \).

In general, a least square criterion is used in data modeling. The fitting error for parameter \( \alpha \) can be expressed as

\[
\chi^2_t = \sum_{k=1}^{N^j_t} f(x_k; \alpha)^2,
\]

but this measure is not the sum of the squared distance between the observed points and the curve. Normalizing \( f(x_k; \alpha) \) by its gradient magnitude produces a function to which the first order approximation is the distance between the point and the curve.

Let

\[
\varepsilon_k = \frac{f(x_k; \alpha)}{||\nabla f(x_k; \alpha)||}
\]

\(
\chi^2 = \sum_{k=1}^{N^j_t} \varepsilon_k^2
\)

\( \frac{1}{\chi^2} \) can be regarded as a weighted version of \( \chi^2 \). It is computationally expensive to minimize \( \chi^2 \) directly. Furthermore, MICH is not the normal least square of boundary points but the central space of the object. Hence, an iterative weighted least square fitting will be needed.

In order to not use the points in \( V^j_t \) that lie outside of the far way from hyperquadric fit or that lie in the central inside area defined by \( f(x) \), we design a weighting function \( \omega(\varepsilon_k; \alpha) \) and a thresholding function \( g(m, \alpha) \) as follows. Let \( m_t \) be the number of points fitted in the \( t \)th iteration, such that \( \varepsilon_k > -g(m_{t-1}, \alpha_{t-1}) \). \( m_0 \) is \( N^j_t \). \( d_o \) denotes an average of half distance between bounding lines which can be calculated from \( \alpha \).

\[
g_t = g(m_t, \alpha_t) = \frac{m_{t-1}}{m_t} \cdot d_o, \quad (t > 0)
\]

\( g(m_0, 0) \) is given as \( d_o \). The threshold is reduced if the number \( m_t \) of data fitted is greater than \( m_{t-1} \) and it increases if \( m_t \) is less than \( m_{t-1} \).

\[
\omega(\varepsilon_k; \alpha_t) = \begin{cases} 
\frac{1}{||\nabla f(x_k; \alpha_t)||^2}, & \text{if } \varepsilon_k > 0 \\
\frac{1}{(1-f(x_k; \alpha_t))^2}, & \text{if } -g(m_{t-1}, \alpha_{t-1}) < \varepsilon_k < 0 \\
0, & \text{otherwise}
\end{cases}
\]

If \( \varepsilon_k > 0 \), \( x_k \) is inside and \( \varepsilon_k < 0 \) is inside \( f(x_k; \alpha_t) < 1 \). All of the inside points must be fitted. Therefore, we enlarge the weights for them as \( f(x_k; \alpha_t) \) approaches to 1, which means that the fitting data point goes too far inner area of hyperquadrics. The outside points far away from hyperquadrics will be discarded by a thresholding function \( g_{t-1} \). We prefer a large convex part within \( X^j_t \), in other words, a large number of data points. Hence, the weighted square error is divided by \( m_t \).

\[
\varepsilon^2_t = \frac{1}{m_t} \sum_{k=1}^{N^j_t} \omega(\varepsilon_k; \alpha_t)f(x_k; \alpha_t)^2
\]

Using this measure of error, we can obtain parameters of MICH for \( H^j_t \) by finding a minimum of the following iterative algorithm. Since \( f(x, \alpha) \) is nonlinear, we adopt the Levenberg-Marquardt optimization method[7] in order to minimize \( \varepsilon^2_t \) by adjusting the parameters of \( f(x_k; \alpha) \). Let \( \alpha^0 \) be a set of initial parameters of \( s \) terms which are selected among the initial \( n \) terms \( (s = 2, \ldots, n) \). The method of extracting several initial terms will be explained in the section 4.3.

Algorithm FindMICH(input: a set of \( \alpha^0 \), \( N^j_t \) fitting points output: \( \alpha^f \))

for all possible combination of \( \alpha^s \) \((s = 2, \ldots, n)\)

1. Let \( f(x; \alpha^s) \) be an initial guess for \( H^j_t \).
2. With \( \alpha^f_t \), calculate \( \varepsilon^2_t \).
3. Find \( \alpha^t+1 \) that minimizes \( \varepsilon^2_{t+1} \).

\[
\varepsilon^2_{t+1} = \frac{1}{m_{t+1}} \sum_{k=1}^{N^j_t} \omega(\varepsilon_k; \alpha^t_{t+1})f(x_k; \alpha^t_{t+1})^2
\]

4. if \( \varepsilon^{t+1} < \varepsilon_t \) and \( m_{t+1} \geq m_t \) then goto step 2
else save \( \alpha^t \), let \( \alpha^t \) be another combination, and goto step 1
Figure 1: Result of F18 shape: (a) Input image (b) Selected fitting points (c) Touched line segments (d) Decomposed parts

4.3. Initial Guess

The initial guess of $f(x)$ can be given as an ellipse from the points in $M_k^i$. From the centroid, maximum axis length, minimum axis length, and orientation, we can calculate parameters of the following ellipse:

$$(a_1x + b_1y + d_1)^2 + (a_2x + b_2y + d_2)^2 = 1.$$ 

This initial ellipse is fitted with boundary points of $X_i$ by the algorithm FindMICHI in order to adjust size, orientation, and center.

The number of local minima depends on the complexity of the shape. Most minimization methods are attracted to the nearest local minimum and may not reach the global minimum. Furthermore, data points to be optimized are changed at each iteration step. The initial guess thus becomes very important to find the global minimum. The best guess will be the line segments which touch the ellipse. Such line segments can be easily extracted from a polygonal approximation of boundary points $(X_i)_{4k}$. Of course, the length of the line segment should be similar with the half axis length of the ellipse at least and not orthogonal to a tangent of the ellipse at the touch point. We illustrate such line segments of wings in Figure 1. (c). We can make several initial guesses by the combinations of such line segments and the ellipse. One line segment generates one term of hyperquadrics. We can rewrite each term

$$\frac{a_ix + b_iy + d_i}{s_i \sqrt{a_i^2 + b_i^2}}$$

where $s_i$ is half of the distance between the two bounding lines[3]. If the line segment has a parallel line segment which touches the ellipse, $s_i$ would be half of the distance between them and they are merged into one term of hyperquadrics. If not, we set $s_i$ to the distance from a center of the ellipse and to the line segment. The initial value for an exponent of these new terms is set to 8. When the new term has similar slope with the axis of ellipse, the new term is discarded and the value of an exponent for the matched axis term of ellipse is raised to 4.

4.4. Constraints

In general, we can fit most convex shapes with less than 7 terms. Hence we limit the number of terms of the initial guess to 7 including 2 terms of the ellipse. Line segments whose length is smaller than the smallest length of the five largest length line segment are not considered. Then, we try to find a hyperquadrics that has a minimum $e^2$ among all combinations of the initial terms.

Since we try to find a maximal convex shape, all exponents of the hyperquadric terms should be greater than one. High order exponent terms do not change the shape much but cause an instability in computation. Hence, we limit $\gamma_i < 40$ as Han[6] used. We merge the collinear line segments, because the combination of collinear terms results in a very thin strip.

5. EXPERIMENTAL RESULTS

We tried the technique on 2 dimensional images that include a natural object and a man-made object. As shown in Figure 1, we have a good result for F18 aircraft shape using only a few terms of hyperquadrics. Especially, the wings are well fitted with 3 hyperquadric terms owing to 4 touching line segments. The exponents of 3 terms are 1.2, 4.6, and 5.2 respectively. The number of decomposed parts is 10. All parts are described by 2 or 3 terms except for the back body that has 4 terms. The exponents of 4 terms are 1.5, 5.1, 36.9, and 38.8 respectively. After fitting, in general, the exponent of the initial ellipse tends to have lower values, and the exponent of the touching line terms tends to have the higher values.
Figure 2: Input images

Figure 3 shows the results for the input images in Figure 2. We can obtain the decomposed parts that correspond to human intuition. One leg of the goose is missing since the thickness of it is less than the size of structuring element. Each parts approximate the input shape very well. A small part of the decomposed shape can be removed if we raise the limit of $n_i$ in Eq (1) from 1 to 2.

6. CONCLUSIONS

In this paper, we present a new hybrid method to recursively decompose 2D object into convex parts, each of which are described by hyperquadrics. A mathematical morphology is used in order to find an appropriate set of points to be used in the weighted least square fitting at each step. Also, we find the higher order terms of the hyperquadrics from the line segments which are in a polygonal approximation and which touch the fitted ellipse.

From our results, we see that hyperquadrics can well describe shape parts using only a few terms and have a greater description power than the superquadrics[2]. Also it does not produce small false parts and it is not affected by rotation in comparison with morphological approach[3]. For the further study, this method would be extended to 3D range data. Also, the minimum number of terms in the fitting could be obtained by some information measures such as a minimum de-

scription length.

7. REFERENCES


