Corner Detection Using the MAP Technique

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Abstract
This paper describes a corner detection method that obtains maximum-aposteriori estimates for the corner location in a given sequence of points. We model an ideal corner as the intersection of two ideal line segments. The perturbations on the sample points in a given line segment are assumed to be i.i.d Gaussian random variables of zero mean and variance $\sigma^2$. Further, the perturbations on the points are assumed to be orthogonal to the ideal line. The paper discusses the theory of the corner detector and an algorithm that extends the basic theory to handle multilinear segment arcs. Experiments were conducted according to a specific protocol and performance curves showing the location error versus the noise variance, the included corner angle, and the arc length, are provided. Performance characterization of the corner detector is also performed by plotting the false alarm rate and the misdetection rate versus the context window length and included corner angle. It is shown that the experimental results match the theoretical error propagation.

1 Introduction
This paper describes a corner detector based on the maximum a posteriori (MAP) technique and Bayes theory to estimate corner locations on a given arc segment. The method can also be applied to polygonal linear approximation. We model an ideal corner as the intersection point of two straight lines. The corner detection model incorporates the prior distributions for corner model parameters. The detection procedure is extended to handle digital arc sequences with multiple corners by sliding a two-line-segment corner detector over the entire sequence iteratively to estimate corner positions and their related line parameters. The perturbations on the sample points in a given line segment are assumed to be i.i.d Gaussian random variables of zero mean and variance $\sigma^2$. Further, the perturbations on the points are assumed to be orthogonal to the ideal line. The paper discusses the theory of the corner detector, the theoretical analysis for the location error and experimental protocol used to evaluate the performance of the algorithm.

2 Models and Algorithm
The ideal corner is the intersection point of two ideal line segments and in the continuous domain these two lines are specified by the equations: $r\cos\theta_1 + c\sin\theta_1 - \rho_1 = 0$ and $r\cos\theta_2 + c\sin\theta_2 - \rho_2 = 0$. The quantities in the expression $\theta_1, \theta_2, \rho_1, \rho_2$ can be derived from the coordinates of three points: $(r_1, c_1)$, the starting point in line 1, $(r_2, c_2)$, the intersection point of lines 1 and 2, and $(r_3, c_3)$, the end point of line 2. The line segments are sampled to obtain a discrete sequence of points: $S = \{(r_i, c_i) | i = 1, ..., I; (r_i, c_i) \in Z_R \times Z_C\}$, where $Z_R \times Z_C$ is the image domain, and $I$ is the number of points. An observed sequence of points $S$ is assumed to be obtained by individually perturbing points $(r_i, c_i)$ with i.i.d Gaussian samples with zero mean and standard deviation $\sigma$. Each point has a unique orientation specified by the orientation of the line segment in which the point belongs. The perturbations in the points are assumed to be introduced in the direction perpendicular to its orientation. Perturbations on the two line segments can be expressed by: $r_i = r_i + \eta_i \cos\theta_1; c_i = c_i + \eta_i \sin\theta_1; i = 1, ..., k; r_i = r_i + \eta_i \cos\theta_2; c_i = c_i + \eta_i \sin\theta_2; i = k + 1, ..., I$, where $\eta_i \sim N(0, \sigma^2)$ and the ideal breakpoint is assumed to be at index $k$.

Prior Distributions – Assumptions
The parameters describing the ideal corner are $\theta_1, \theta_2, \rho_1, \rho_2, k$, and $I$. Certain assumptions on the nature of the prior distributions are made in this work. These assumptions are discussed here. The index $k$ and parameters $(\theta_1, \rho_1)$ $(\theta_2, \rho_2)$ are independent of $\sigma$. Furthermore, the index $k$ is independent of the line parameters $(\theta_1, \rho_1)$ $(\theta_2, \rho_2)$, and these line parameters are independent of the number of points $I$, therefore the prior probability distribution $P(k, \theta_1, \rho_1, \theta_2, \rho_2 | I)$ can be written as $P(\theta_2 | \theta_1)P(\rho_1 | \theta_1)P(\theta_1)P(\rho_1)P(k | I)$. We assume the index $k$ to be uniformly dis-

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distributed. That is:
\[ P(k | I) = \begin{cases} 
K_k = 1/I - 2, & 2 \leq k \leq I - 1, \\
0, & \text{otherwise.}
\end{cases} \]

Assume \( \theta_1 \) to be uniformly distributed in the domain \([0, 2\pi)\), i.e. \( P(\theta_1) = 1/2\pi \). We assume that the image domain is a square, i.e. \( Z = Z_R = Z_C \) and centered at \((1/2 Z, 1/2 Z)\). We also assume that \( \rho_1 \) is uniformly distributed in the interval \([0 \leq \rho_1 < Z]\). Thus:
\[ P(\rho_1 | \theta_1) = 1/Z. \]
Similarly, we assume that \( P(\rho_2) = 1/Z, [0 \leq \rho_2 < Z]\). Let \( \theta_{12} = | \theta_2 - \theta_1 | \) be defined in the domain \([0, \pi]\). We assume the probability distribution of \( \theta_{12} \) to be a truncated form of a Von Mises distribution with mean angle \( \pi/2 \). That is:
\[ P(\theta_{12}) = K_1e^{K_2\sin(\theta_{12})}, \]
where \( K_1 \) is a scale factor and \( K_2 \) is a precision parameter for the angle.

**Problem & Solution**

Given a sequence of observations \( \hat{S} \), the problem is to find the maximum a posteriori estimates for the index of the pixel corresponding to the corner location \((k^*)\) and the line parameters \((\theta_1^*, \rho_1^*, \theta_2^*, \rho_2^*)\). That is:
\[ \arg \max_{(k, \theta_1, \rho_1, \theta_2, \rho_2)} P(k, \theta_1, \rho_1, \theta_2, \rho_2 | \hat{S}, \sigma, I). \]

It can be shown that the logarithm of the aposteriori probability is given by
\[ \log K - \frac{1}{2\sigma^2} \sum_{k=1}^{\hat{S}} (\hat{r}_k \cos \theta_1 + \hat{c}_k \sin \theta_1 - \rho_1)^2 - \frac{1}{2\sigma^2} \sum_{i=k+1}^{I} (\hat{r}_i \cos \theta_2 + \hat{c}_i \sin \theta_2 - \rho_2)^2 + K_2 \sin(\theta_2 - \theta_1), \]
where \( K = \log K_1 - \log(2\pi) - 2 \log Z - \log(I - 2) - I \log(2\pi \sigma). \) Taking partial derivatives with respect to the parameters \( \theta_1, \rho_1, \theta_2, \rho_2 \) and setting the result to zero provides a system of nonlinear equations. The initial solution to the nonlinear equations is obtained by using maximum-likelihood estimation. A more precise solution is obtained by using gradient search. We do not provide the algorithm details here due to lack of space and more details can be found in [11].

In reality, we may be provided with pixel chains that contain more than one corner. We use the two-line-segment corner detector with a certain context window length and slide the window to perform detection on the entire arc. The procedure starts by examining the first \( I \) (assuming that \( I \) is the context window length) pixels of the given pixel chain. If a corner is detected, the corner detector moves to the next context window starting at the pixel next to the detected corner pixel. If the corner is not detected, the detector is moved along the sequence by a fixed step size (usually one pixel) and the detector is reapplied. We repeat this procedure until the tail of the context window reaches the end point of the given digital arc. The presence or absence of a corner within a given window is determined by a probability threshold. It can be easily shown that the value of the objective function is chi-square distributed (with \( I \) degrees of freedom) when there is no corner (i.e.) \( \theta_1 = \theta_2 \). If \( T \) is the threshold on the value of the objective function, the probability of false alarm is given by:
\[ \text{Prob}(X_1 < T_p) = \alpha_T. \]
Hence, we choose a confidence level \( \alpha_T \) and set the threshold \( T_p \) so that
\[ \text{Prob}(X_1 < T_p) = \alpha_T. \]
In reality, the objective function is evaluated at the estimates \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \) and the minimum value is compared against \( T_p \) to determine if the breakpoint is significant or not.

### 3 Experimental Protocol and Results

In this section we describe the experimental protocol used to characterize the performance of the operator. Specifically, we measure: the precision of the estimated corner location as a function of the detector parameters, noise standard deviation and ideal corner model parameters, and the false alarm and misdetect characteristics. The input parameters to the corner detector are the context window length \( cwi \), the estimated standard deviation of the noise \( \sigma \), and the confidence coefficient \( \alpha_T \).

#### Two Line Segment Arc Generation

A perturbed two line segment arc, \( \hat{S} = (r_i, c_i); i = 1, ..., I \), is generated in three steps. First, generate the coordinates of the points \((r_i, c_i)\) on the digital line segments by discretizing the continuous line segments described with parameters \((r_1, c_1), L_1, \theta_1, \theta_1, \theta_2, \theta_2, L_2\). Second, generate the sequence of samples \( \{m_i; i = 1, ..., I\} \), where each \( m_i; i = 1, ..., I \) is an independent random sample coming from a Gaussian distributed random variable with zero mean and a standard deviation \( \sigma \). Third, obtain a perturbed sequence of the arc segment by using the relations:
\[ < (r_i + m_i \cos \theta_1, c_i + m_i \sin \theta_1); i = 1, ..., I >, \]
and
\[ < (r_i + m_i \cos \theta_2, c_i + m_i \sin \theta_2); i = I^2 + 1, ..., I >. \]

#### Location Error Measurement

First, we measure the location errors versus noise standard deviation, the included corner angle and the arc sequence length. We apply the corner detector to synthetically generated two-line-segment sequences and obtain the distance between the true corner and the detected corner. For this experiment, the context window length is equal to the length of the sequence, \( \sigma \) is systematically set up, and \( \alpha_T \) is not used. We let \( \theta_{12} = 90^\circ \) and \( L_1 = L_2 = 50 \) units. For each \( \sigma \in \{0, 0.2, 0.4, ..., 5.0\} \) and for all of \( \theta_1 \in \{0^\circ, 1^\circ, ..., 359^\circ\} \), we generate 10 sequences of
two-line-segment arcs. There are 360*10 runs, defined as \( N_{run} \), for each \( \sigma \). We apply the corner detector and obtain the root-mean-square error of the distance and the variance of this distance. Figures 1(a) and 1(b) are respectively the root-mean-square location error and the root-mean-square variance of the location error versus the noise standard deviation. It indicates that the error linearly increases as the noise increases and the variance of the error quadratically increases as the noise increases. It indicates that the theoretical computations and the experimental results are consistent.

We choose \( \theta_{12} \) from the set \{10°, 20°, ..., 170°\} and set \( \sigma = 1.0 \) to obtain plots shown in Figure 1(c), which illustrates the root-mean-square location error versus \( \theta_{12} \), and indicates that the detection has the tendency of having smaller error for 90° corner angle and larger error for corner angles away from 90°. In addition, the rather flat region around 90° corner angle indicates that the algorithm is more stable over a large range of corner angles. Figure 1(d) shows plots of the root-mean-square location error versus the arc length. The result indicates that the algorithm is stable with different arc lengths.

**False alarm/Misdetect Characteristics**

We evaluate the performance of the detector by plotting its false alarm rate and misdetection rate versus the context window length \( cwl \), the included corner angle \( \theta_{12} \) and the distance threshold \( d_0 \) which is a special parameter used during performance test. Here, \( cwl \) is chosen smaller than the sequence length, \( \sigma \) is systematically set up, and \( \sigma_{\text{f}} = 0.9 \). The false alarm rate is defined as the probability of noncorners being detected as corners, ie. \( \text{Prob} \) (detected as corners | noncorners), and the misdetection rate is defined as the probability of true corners not being detected as corners, ie. \( \text{Prob} \) (not detected as corners | true corners). Define a radius of \( d_0 \), called the distance threshold, centered at a true corner. If no point exists within the circle defined by the given radius, a misdetection happens. If the detected corner does not fall into any circular region of radius \( d_0 \) centered at a true corner, this detection is claimed as a false alarm.

We set \( \theta_{12} = \pi/2 \), \( L_1 = L_2 = 50 \) units, and \( d_0 = 3 \).

For each context window length \( cwl \in \{3, 4, \ldots, 70\} \), and \( \theta_1 \in [0, 2 \pi] \) in increments of 1 degree, we generate 10 sequences of two line segment arcs. We apply the corner detector and plot the false alarm and misdetection rates. Figure 2(a) is the false alarm rate versus the context window length and it implies that the developed algorithm is more stable if a context window length is large enough to contain sufficient information for the estimation. Figure 2(b) is the misdetection rate versus the context window length and it shows that the rate drops linearly when the window length increases. Figure 2(c) and 2(d) show plots of the false alarm rate and the misdetection rates as a function of the included corner angle when \( cwl = 2 \times 50 \) units. They show that the algorithm has small false alarm rate and misdetection rate around the 90°.

4 Conclusions

We have discussed a corner detector that is based on MAP estimation. We have evaluated the performance on synthetic data and have provided theoretical and empirical curves of performance. The experimental results showed that the theoretical and experimental results are consistent, and that this method is not very sensitive to random perturbations, is robust, stable and precise. More rigorous theoretical analysis of this operator is being done and future work will involve the theoretical and empirical comparison of our algorithm with traditional methods. We are also in the process of evaluating performance of our algorithm on a large collection of aerial images.

**References**


Figure 1: (a) Location error versus $\sigma$, (b) Variance of the location error versus $\sigma$, (c) Location error versus the included corner angle, and (d) Location error versus the arc length.

Figure 2: False alarm and Misdetection Characteristics. Figures 2 (a)-(d) and 3(a)-(b).