The Electromagnetic Spectrum

**THE ELECTROMAGNETIC SPECTRUM**

- **Penetrates Earth Atmosphere?**
  - Y | N | Y | N

- **Wavelength (meters)**
  - Radio: $10^3$
  - Microwave: $10^{-2}$
  - Infrared: $10^{-5}$
  - Visible: $0.5 \times 10^{-6}$
  - Ultraviolet: $10^{-8}$
  - X-ray: $10^{-10}$
  - Gamma Ray: $10^{-12}$

- **About the size of...**
  - Buildings
  - Humans
  - Honey Bee
  - Pinpoint
  - Protozoans
  - Molecules
  - Atoms
  - Atomic Nuclei

- **Frequency (Hz)**
  - $10^4$ to $10^{20}$

- **Temperature of bodies emitting the wavelength (K)**
  - 1 K, 100 K, 10,000 K, 10 Million K
Color

Definition

Color is a visual sensation related to the wavelength distribution of the light energy hitting the retina of the eye.

When that wavelength distribution is dominated by wavelengths centered in the

- 425nm band, the eye sees violet
- 470nm band, the eye sees blue
- 525nm band, the eye sees green
- 575nm band, the eye sees yellow
- 665nm band, the eye sees red
Most of the scientists in Newton’s day thought that the prisms were merely adding the color to the light, rather than separating the colors from the light.
Refraction

The speed of light in a medium changes with the wavelength of the light.

\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}
\]

Denser mediums bend light toward the normal.
Refractive Index As a Function of Wavelength for Different Kinds of Glass
Newton’s Two Prism Experiments
Newton was the first one to prove that white light is made of all the colors of the rainbow. The proof: He placed a second prism upside-down in front of the first prism. The band of colors that was split up by the first prism was combined again by the second prism into white sunlight.
Retina
There are two types of photoreceptors in the human retina, rods and cones.

**Rods**
- Rods are responsible for vision at low light levels (scotopic vision)
- Have a low spatial acuity
- Do not participate in color visual sensation
- 100 million rods

**Cones**
- Cones are active at higher light levels (photopic vision)
- Are capable of color vision
- Have a high spatial acuity.
- Three types of cones: called S-cone, M-cone, L-cone
- The central 300\(\mu m\) of the fovea is composed totally of cones
- Fovea has 1.5° – 2° visual angle
- 5 million cones
Rods and Cones

3.4 THE SPATIAL MOSAIC OF THE HUMAN CONES. Cross sections of the human retina at the level of the inner segments showing (A) cones in the fovea, and (B) cones in the periphery. Note the size difference (scale bar = 10 μm), and that, as the separation between cones grows, the rod receptors fill in the spaces. (C) Cone density plotted as a function of distance from the center of the fovea for seven human retinas; cone density decreases with distance from the fovea. Source: Curcio et al., 1990.
Rods and Cones
Cone Relative Sensitivity

Data from Stockman and Sharpe Colour Vision Laboratory
www.cvrl.org
Color Response

- $\lambda$ wavelength
- $p_S(\lambda) \geq 0$ cone-S sensitivity for short wavelengths
- $p_M(\lambda) \geq 0$ cone-M sensitivity for medium wavelengths
- $p_L(\lambda) \geq 0$ cone-L sensitivity for long wavelengths
- $e(\lambda) \geq 0$ spectral radiance emitted at wavelength $\lambda$

Response-S
$$r_S = \int e(\lambda) p_S(\lambda) d\lambda$$

Response-M
$$r_M = \int e(\lambda) p_M(\lambda) d\lambda$$

Response-L
$$r_L = \int e(\lambda) p_L(\lambda) d\lambda$$
CIE 1978 Eye Sensitivity

Shows the overall eye sensitivity as a function of wavelength. The eye is much more sensitive to greens than blues or reds.
Peacock Mantis Shrimp

3-18cm; forearms can acceleration similar to .22 caliber bullet, 340 pounds force per strike. Can see from infrared through ultraviolet. Eight different types of photoreceptors. Can distinguish circularly polarized light.
Spectral Radiance emitted depends on the spectral radiance of the light source and the percent reflection of the illuminated surface.
The spectral radiance is the component-wise product of the reflectance times the illuminance. Reflectance can depend on the angle of illumination.
Multiple Conditions Have Appearance as Yellow

Seeing yellow can mean

- Wavelengths between $577 - 597\,\mu m$
- Any Balanced Mixture of Red and Green
  - 700nm and 530nm
  - infinitely many other mixtures

A yellow spotlight causes the illuminated surface to appear yellow.

To the eye-brain system, there is no difference between yellow light and overlapping red and green light.
Two objects with different spectral reflectance functions can appear to be the same color in one light and appear to have different colors in another light.
Multiple Conditions Can Have Same Color Appearance

The appearance of color is determined by the response of the cones: \((r_S, r_M, r_L)\)

\[
\text{Response-S } r_S = \int e(\lambda) p_S(\lambda) d\lambda
\]

\[
\text{Response-M } r_M = \int e(\lambda) p_M(\lambda) d\lambda
\]

\[
\text{Response-L } r_L = \int e(\lambda) p_L(\lambda) d\lambda
\]

If for two instances

\((r_{1S}, r_{1M}, r_{1L}) = (r_{2S}, r_{2M}, r_{2L})\)

the instances have the same color appearance.
Integration

\[ I = \int_{a}^{b} f(x) \, dx \]

Let

\[ x_0 = a \]
\[ x_n = a + n\Delta = x_{n-1} + \Delta \]
\[ x_{10} = b \]
\[ R_n = x_n\Delta \]

\[ I \approx \sum_{n=1}^{10} f(x_n) \Delta \]
Convex Combinations

If two spectral radiances produce the same color appearance, then their convex combination produces that color appearance. Suppose

\[ r_S \approx \sum_{n=1}^{N} e_1(\lambda_n) p_S(\lambda_n) \Delta \]
\[ r_S \approx \sum_{n=1}^{N} e_2(\lambda_n) p_S(\lambda_n) \Delta \]

Let \( 0 \leq \alpha \leq 1 \). Then

\[ 0 \leq \alpha e_1(\lambda) + (1 - \alpha) e_2(\lambda) \quad \text{A Convex Combination of } e_1 \text{ and } e_2 \]

\[ r'_S \approx \sum_{n=1}^{N} [\alpha e_1(\lambda_n) + (1 - \alpha) e_2(\lambda)] p_S(\lambda_n) \Delta \]

\[ = \alpha \sum_{n=1}^{N} e_1(\lambda_n) p_S(\lambda_n) \Delta + (1 - \alpha) \sum_{n=1}^{N} e_2(\lambda_n) p_S(\lambda_n) \Delta \]

\[ = \alpha r_S + (1 - \alpha) r_S = r_S \]
Sum Approximations

Define

\[ e(\lambda_n) = e_n \]
\[ p_{Sn} = p_S(\lambda_n) \Delta \]
\[ p_{Ln} = p_L(\lambda_n) \Delta \]
\[ p_{Mn} = p_M(\lambda_n) \Delta \]

Then

\[ r_S = \sum_{n=1}^{N} e_n p_{Sn} \]
\[ r_M = \sum_{n=1}^{N} e_n p_{Mn} \]
\[ r_L = \sum_{n=1}^{N} e_n p_{Ln} \]

\[
\begin{pmatrix} r_S \\ r_M \\ r_L \end{pmatrix} = \begin{pmatrix} p_{S1} & \cdots & p_{SN} \\ p_{M1} & \cdots & p_{MN} \\ p_{L1} & \cdots & p_{LN} \end{pmatrix} \begin{pmatrix} e_1 \\ \vdots \\ e_N \end{pmatrix}
\]

\[ r = Pe \]
Let  

\[ e^{N \times 1} = \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{pmatrix} \]

Then  

\[ e^{N \times 1} \geq 0 \]

means  

\[ e_n \geq 0, \quad n = 1, \ldots, N \]
Convex Combinations

If two spectral radiances produce the same color appearance, then their convex combination produces that color appearance.

\[ r^{3 \times 1} = P^{3 \times N} e^{n \times 1} \]

Suppose \( e_1 \geq 0 \) and \( e_2 \geq 0 \) satisfy

\[
\begin{align*}
  r &= Pe_1 \\
  r &= Pe_2
\end{align*}
\]

Let \( 0 \leq \alpha \leq 1 \). Then,

\[
\begin{align*}
  \alpha e_1 + (1 - \alpha) e_2 & \geq 0 \\
  P(\alpha e_1 + (1 - \alpha) e_2) &= \alpha Pe_1 + (1 - \alpha) Pe_2 \\
  &= \alpha r + (1 - \alpha) r \\
  &= r
\end{align*}
\]
Let $r^{3\times 1}$ and $P^{3\times N}$ be given and let $e_0^{N\times 1} \geq 0$ be any spectral radiance vector satisfying $r = Pe_0$, $N > 3$. Consider the solution set

$$S = \{ e \mid Pe = r \text{ and } e \geq 0 \}$$

The set $K = \{ a \mid Pa = 0 \}$ is called the Kernel of $P$. Then,

$$S = \{ e \mid e = e_0 + a, \ a \in K \text{ and } e \geq 0 \}$$

$S$ is the positive quadrant of $\{ \text{the subspace } K \text{ translated by } e_0 \}$ $S$ has infinitely many elements.

There are many distinctly different spectral radiance emittances that we see as the same color.
There are many systems for specifying color.

- RGB
- HSB
- HSL
- CIE
- CIELa*b*
- Munsell
Can it be possible for a mixture of Red, Green, and Blue to have the appearance of any given color?
Let $r^{3 \times 1}$ be a given cone response vector. $r$ corresponds to a particular color appearance.

Suppose that

- Blue has spectral radiance at a single wavelength $\alpha$ at intensity $e_\alpha$
- Green has spectral radiance at a single wavelength $\beta$ at intensity $e_\beta$
- Red has spectral radiance at a single wavelength $\gamma$ at intensity $e_\gamma$

Then the spectral radiance vector $e$ is all 0’s with the exception of components $\alpha$, $\beta$, and $\gamma$

$$
\begin{pmatrix}
  r_S \\
  r_M \\
  r_L
\end{pmatrix} = \begin{pmatrix}
  p_{S\alpha} & p_{S\beta} & p_{S\gamma} \\
  p_{M\alpha} & p_{M\beta} & p_{M\gamma} \\
  p_{L\alpha} & p_{L\beta} & p_{L\gamma}
\end{pmatrix} \begin{pmatrix}
  e_\alpha \\
  e_\beta \\
  e_\gamma
\end{pmatrix} = r
$$

$$
\begin{pmatrix}
  e_\alpha \\
  e_\beta \\
  e_\gamma
\end{pmatrix} = \begin{pmatrix}
  p_{S\alpha} & p_{S\beta} & p_{S\gamma} \\
  p_{M\alpha} & p_{M\beta} & p_{M\gamma} \\
  p_{L\alpha} & p_{L\beta} & p_{L\gamma}
\end{pmatrix}^{-1} \begin{pmatrix}
  r_S \\
  r_M \\
  r_L
\end{pmatrix} = r
$$

If $(e_\alpha, e_\beta, e_\gamma) \geq 0$ then the color specified by $(r_S, r_M, r_L)$ can be produced by a mixture of red, green and blue.
Can RGB Make An Arbitrary Color?

Let

- \( e_{\text{red}}^{N \times 1} \) be the spectral radiance of a given red source that produces cone response \( a^{3 \times 1} = Pe_{\text{red}}^{N \times 1} \)
- \( e_{\text{green}}^{N \times 1} \) be the spectral radiance of a given green source that produces cone response \( b^{3 \times 1} = Pe_{\text{green}}^{N \times 1} \)
- \( e_{\text{blue}}^{N \times 1} \) be the spectral radiance of a given blue source that produces cone response \( c^{3 \times 1} = Pe_{\text{blue}}^{N \times 1} \)
- \( r \) be a cone response vector specifying a given color appearance

Does there exist mixing coefficients \( \alpha, \beta, \gamma \geq 0 \) such that

\[
\alpha e_{\text{red}} + \beta e_{\text{green}} + \gamma e_{\text{blue}}
\]

produces the color \( r \)?
Can RGB Make An Arbitrary Color

Does there exist mixing coefficients $\alpha, \beta, \gamma \geq 0$ such that

$$\alpha e_{\text{red}} + \beta e_{\text{green}} + \gamma e_{\text{blue}}$$

produces the color $r$?

$$r = P(\alpha e_{\text{red}} + \beta e_{\text{green}} + \gamma e_{\text{blue}})$$
$$= \alpha Pe_{\text{red}} + \beta Pe_{\text{green}} + P\gamma e_{\text{blue}}$$
$$= \alpha a + \beta b + \gamma c$$

$$= \begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}^{-1}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} a & b & c \end{pmatrix}^{-1} r$$
The perceived color attribute of a visual sensation according to which an area appears to be similar to one of the perceived colors: red, yellow, green, and blue, or to some combination of two of them.

It can be measured in angular degrees counter-clockwise around the cylinder:
- Red 0°
- Yellow 60°
- Green 120°
- Cyan 180°
- Blue 240°
- Magenta 300°

The Hue specified by \((R, G, B)\) is the same hue as specified by \(\lambda(R, G, B)\) for \(0 \leq \lambda \leq 1\).
Every point can be specified as (angle, radius).
Each color can be represented as a point on a polar (Hue, Chroma) plot.

- The Hue is the angle coordinate.
- The Chroma is the radial coordinate.
RGB values as a function of Hue with Chroma = 1.
RGB values as a function of Hue with Chroma = .75
RGB values as a function of Hue with Chroma = .50
Chroma = .25

RGB values as a function of Hue with Chroma = .25
Hue Chroma in Polar Plot

Chroma on the outside ring is 1. Chroma decreases by 15% as the ring moves inward toward the center.
Brightness

The attribute of a visual sensation according to which an area appears to emit more or less light.

Brightness can be measured in percent from black (0) to white (100) or in [0, 1].

Brightness in HSB is defined by \( \max\{R, G, B\} \).

Colors with the same HSB Brightness do not have the same luminance.

At 0% brightness, hue is meaningless.
Hue and Brightness

Brightness decreases by .1 as each annulus gets closer to the center.
Brightness decreases by 15% as each annulus gets closer to the center.
Saturation indicates the degree to which the hue differs from a neutral gray.

Saturation values run from 0%, which is no color saturation, to 100%, which is the fullest saturation of a given hue at a given percentage of brightness.

In HSB, Saturation is defined by

\[
\text{Saturation} = \frac{\text{max}\{R,G,B\} - \text{min}\{R,G,B\}}{\text{max}\{R,G,B\}}
\]

At 0% saturation, hue is meaningless.

In the above strips, Saturation goes from 1, .6, .25, .15 with brightness held at 1.

In the bottom strips, Saturation goes from 1, .6, .25, .15 with brightness held at .5.
Let $R, G, B$ the the values of Red, Green, and Blue. The brightness $b$ is given by

$$b = \max\{R, G, B\}$$

The saturation $s$ is given by

$$s = \frac{\max\{R, G, B\} - \min\{R, G, B\}}{\max\{R, G, B\}}$$

The saturation of $(R, G, B)$ is the same as the saturation of $(\lambda R, \lambda G, \lambda B)$
The saturation is reduced by 15% as the rings move inward to the center. The brightness is fixed at 1.
Motivation for the HSB System

Suppose a color is specified by \((R, G, B)\).

- \(c_{\text{max}} = \max\{R, G, B\}\)
- \(c_{\text{min}} = \min\{R, G, B\}\)
- \(c_{\text{min}}(1, 1, 1)\) represents the amount of white added to the base color
- \((R, G, B) = c_{\text{min}}(1, 1, 1) + (R - c_{\text{min}}, G - c_{\text{min}}, B - c_{\text{min}})\)

Suppose \(c_{\text{max}} = R\) and \(c_{\text{min}} = B\).

\[(R, G, B) = c_{\text{min}}(1, 1, 1) + (c_{\text{max}} - c_{\text{min}}, G - c_{\text{min}}, 0)\]
\[= c_{\text{min}}(1, 1, 1) + (c_{\text{max}} - c_{\text{min}})(1, \frac{G - c_{\text{min}}}{c_{\text{max}} - c_{\text{min}}}, 0)\]
\[= c_{\text{min}}(1, 1, 1) + c_{\text{max}} \left[\frac{c_{\text{max}} - c_{\text{min}}}{c_{\text{max}}}\right] \left(1, \frac{G - c_{\text{min}}}{c_{\text{max}} - c_{\text{min}}}, 0\right)\]

- \(c_{\text{max}}\) is the HSB Brightness
- \(\frac{c_{\text{max}} - c_{\text{min}}}{c_{\text{max}}}\) is the HSB Saturation
The HSB Cylinder

HSB: Hue Saturation Brightness is same as HSV: Hue Saturation Value
Hue: 0°; Saturation decreases horizontally by 20%; Brightness decreases vertically by 20%
HSB Cylinder Slab

Hue: 15°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 30°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 45°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 60°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 75°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
HSB Cylinder Slab

Hue: 90°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 105°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
HSB Cylinder Slab

Hue: 120°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 135°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 150°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 165°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 180°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 195°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 210°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 225°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 240°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 255°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 270°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 285°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 300°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 315°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 330°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 345°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Hue: 360°; Saturation decreases horizontally by 20% Brightness decreases vertically by 20%
Ring saturation decreases by 20% toward the center. Brightness is 1.
Ring Saturation decreases by 20% toward the center. Brightness is 1.

- Colors with the same brightness value are not perceptually the same brightness
- Fully saturated blue is perceived to be darker than fully saturated red or green
- Reds are more flashy than the blues
Ring saturation decreases by 20% in each ring toward the center. Brightness is $0.8^0 = 1$. 
Ring saturation decreases by 20% in each ring toward the center. Brightness is $0.8^1 = 0.8$. 
Ring saturation decreases by 20% in each ring toward the center. Brightness is $0.8^2 = 0.64$. 
Ring saturation decreases by 20% in each ring toward the center. Brightness is $0.8^3 = 0.512$. 
Ring saturation decreases by 20% in each ring toward the center.
Brightness is $0.8^4 = 0.4096$. 
Ring saturation decreases by 20% in each ring toward the center. Brightness is $0.8^5 = 0.32768$. 
Ring saturation decreases by 20% in each ring toward the center. Brightness is $0.8^6 = 0.262144$. 
Ring saturation decreases by 20% in each ring toward the center. Brightness is $0.8^7 = 0.2097152$. 
Ring saturation decreases by 20% in each ring toward the center. Brightness is $0.8^8 = 0.16777216$. 
Ring saturation decreases horizontally by 20% in each ring toward the center. Brightness is $0.8^9 = 0.134217728$. 
RGB Color Cube

- Blue: (0, 0, 1)
- Cyan: (0, 1, 0)
- White: (1, 1, 1)
- Black: (0, 0, 0)
- Red: (1, 0, 0)
- Green: (0, 1, 0)
- Magenta: (1, 0, 1)
- Grey scale
Planes Of Constant Brightness

Brightness is here defined as $R+G+B=\text{constant}$. 

![Graphs showing planes of constant brightness at 25%, 50%, and 75% brightness levels.](image)
Wedges Of Constant Hue
Cones Of Constant Saturation

Saturation = 20%

Saturation = 70%
The Sawtooth

For $0 \leq x \leq 360$

$$f(x) = \frac{x}{60} - \lfloor \frac{x}{60} \rfloor$$
HSB to RGB

\[ i(h) = \left\lfloor \frac{h}{60} \right\rfloor \]

\[ f(h) = \frac{h}{60} - i(h) \]

\[ c_{\text{max}} = b \]

\[ c_{\text{min}} = c_{\text{max}}(1 - s) \]

\[ q(h) = c_{\text{max}}(1 - f(h)s) \]

\[ t(h) = c_{\text{max}}(1 - (1 - f(h))s) \]

\[
(R, G, B) = \begin{cases} 
(c_{\text{max}}, t(h), c_{\text{min}}), & i(h) = 0 \\
(q(h), c_{\text{max}}, c_{\text{min}}), & i(h) = 1 \\
(c_{\text{min}}, c_{\text{max}}, t(h)), & i(h) = 2 \\
(c_{\text{min}}, q(h), c_{\text{max}}), & i(h) = 3 \\
(t(h), c_{\text{min}}, c_{\text{max}}), & i(h) = 4 \\
(c_{\text{max}}, c_{\text{min}}, q(h)), & i(h) = 5 
\end{cases}
\]
hsb_to_rgb((float)h,(float)s,(float)b)
    i=(int)(h/60);
    f=h/60-i;
    mx=b;
    mn=mx*(1-s);
    q=mx*(1-f*s);
    t=mx*(1-(1-f)*s);
    switch(i)
        case 0:
            red=mx;
            green=t;
            blue=mn;
            break;
        case 1:
            red=q;
            green=mx;
            blue=mn;
            break;
        case 2:
            red=mn;
            green=mx;
            blue=t;
            break;
        case 3:
            red=mn;
            green=q;
            blue=mx;
            break;
        case 4:
            red=t;
            green=mn;
            blue=mx;
            break;
        case 5:
            red=mx;
            green=mn;
            blue=q;
            break;
    return(red,green,blue);
HSB to RGB

RGB As A Function of HSB: B=.8 S=.75

Value

0.8

0.6

0.4

0.2

0

Hue

0 60 120 180 240 300 360
Let \((R, G, B) \in [0, 1]^3\) be given. Let \(c_{\text{max}} = \max\{R, G, B\}\) and \(c_{\text{min}} = \min\{R, G, B\}\). There are six possibilities:

\[
\begin{pmatrix}
    c_{\text{max}} \\
    c_{\text{min}}
\end{pmatrix} \in \left\{ \begin{pmatrix} R \\ G \end{pmatrix}, \begin{pmatrix} R \\ B \end{pmatrix}, \begin{pmatrix} G \\ R \end{pmatrix}, \begin{pmatrix} G \\ B \end{pmatrix}, \begin{pmatrix} B \\ R \end{pmatrix}, \begin{pmatrix} B \\ G \end{pmatrix} \right\}
\]

The argument is similar for each of the cases. Consider the case

\[
\begin{pmatrix}
    c_{\text{max}} \\
    c_{\text{min}}
\end{pmatrix} = \begin{pmatrix} R \\ G \end{pmatrix}
\]

Then the hue angle \(\theta\) satisfies \(300^\circ \leq \theta \leq 360^\circ\). Whatever the value of \(B\),

\[
360 - \frac{B - c_{\text{min}}}{c_{\text{max}} - c_{\text{min}}} \times 60^\circ = \theta
\]
General Case: $f$

$$f = \begin{cases} \frac{G-B}{R-B} & R \max \{R, G, B\} \quad \frac{B-G}{R-G} & R \min \{R, G, B\} \\ \frac{B-R}{G-R} & G \max \{R, G, B\} \quad \frac{R-B}{G-B} & G \min \{R, G, B\} \\ \frac{G-R}{B-R} & B \max \{R, G, B\} \quad \frac{R-G}{B-G} & B \min \{R, G, B\} \end{cases}$$

$$h = \text{atan2}(\sqrt{3}(G-B), 2R-G-B)$$
HSB: Hue Equivalence Classes

\[ R_0 = \max\{R_0, G_0, B_0\} \text{ and } B_0 = \min\{R_0, G_0, B_0\} \]

- Brightness: \( b \)
- Saturation: \( s \)
- Green Fraction: \( f = \frac{G_0 - B_0}{R_0 - B_0} \)

Equivalence Class:

\[
\begin{bmatrix}
R_0 \\
G_0 \\
B_0
\end{bmatrix}
= \left\{ \begin{pmatrix} R \\ G \\ B \end{pmatrix} \in [0, 1]^3 \right\}
\text{for some } b, s \in [0, 1],
\]

\[
\begin{pmatrix} R \\ G \\ B \end{pmatrix} = b(1 - s) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + bs \begin{pmatrix} 1 \\ f \\ 0 \end{pmatrix}
\]
Hue Equivalence Class

Note that when

\[ f = \frac{G_0 - B_0}{R_0 - B_0} \]
\[ b = R_0 \]
\[ s = \frac{R_0 - B_0}{R_0} \]
\[ b(1 - s) = B_0 \]
\[ bs = R_0 - B_0 \]

\[
\begin{pmatrix}
R_0 \\
G_0 \\
B_0
\end{pmatrix}
= b(1 - s)
\begin{pmatrix}
1 \\
1 \\
1
\end{pmatrix}
+ bs
\begin{pmatrix}
1 \\
1 \\
0
\end{pmatrix}
\]
Proposition

For $0 \leq \lambda \leq 1$,

\[
\lambda \begin{pmatrix} R_0 \\ G_0 \\ B_0 \end{pmatrix} \in \begin{bmatrix} R_0 \\ G_0 \\ B_0 \end{bmatrix}
\]

Proof.

\[
\lambda \begin{pmatrix} R_0 \\ G_0 \\ B_0 \end{pmatrix} = (\lambda b)(1 - s) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + (\lambda b)s \begin{pmatrix} 1 \\ f \\ 0 \end{pmatrix}
\]
HSB is not perceptually uniform when saturation and brightness are held constant.

- With brightness and saturation held constant, the perceived difference between colors is not proportional to their separation angle in hue.

The saturation and brightness are set to 1. The difference in hue is $60^\circ$ between each pair. One pair is perceptually more similar than the other pair.
HSB is not perceptually uniform when saturation and brightness are held constant.

With brightness and saturation held constant, the perceived difference between colors is not proportional to their separation angle in hue.

The saturation and brightness are set to 1. The difference in hue is $60^\circ$ between each pair. One pair is perceptually more similar than the other pair.
Ewald Hering

- German physiologist and psychologist
- A founder of modern visual science
- Lived 1834-1918
- Taught at
  - Josephs-Akademie, Vienna (1865-1870)
  - University of Prague (1870-1895)
  - University of Leipzig (1895)
- Ideas based on color experience
- Postulated three types of color axes
  - Yellow-Blue
  - Red-Green
  - Black-White
- Opponent Color Processing
- Apparently a contradiction to Trichromatic Theory of Young and Helmholtz
Yellow is psychologically just as basic as red, green, or blue.

Dichromats who lack red or green cones cannot see red or green colors but can see yellow.

Most colors of the spectrum seem to shift in hue as they brighten or darken.

These shifts do not happen in blue, green, or yellow colors.
Hering’s Opponent Processes

- red mixed with yellow yields orange
- red mixed with blue yields purple
- red mixed with green never creates the sensation of reddish green or greenish red
- blue mixed with yellow never creates the sensation of bluish yellow or yellowish blue
Hue Circle Explained by Hering’s Opponent Processes

25% blue with 75% red produces crimson red
75% blue with 25% red produces blue violet
50% blue with 50% red produces violet
The chromatic processing behind the retina.
Keep eyes concentrated on black dot for 20 seconds.
Typically genetically inherited through X-chromosome
- Genes that produce the photopigments are on X-chromosome

Missing or Faulty cones
- L-cones: (First) Protanopia, Protanomaly
- M-cones: (Second) Deuteranopia, Deuteranomaly
- S-cones: (Third) Trianopia, Trianopaly

Deficiencies in M-cones or L-cones is commonly called red-green color blindness

Deficiencies in S-cones is commonly called blue-yellow color blindness
Twelve examples of pairs of horizontal blocks between which color blind people may not be able to distinguish.
Color Blindness Prevalence

<table>
<thead>
<tr>
<th>Gender</th>
<th>Protanopia</th>
<th>Protanomaly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>1.00%</td>
<td>1.3%</td>
</tr>
<tr>
<td>Female</td>
<td>0.02%</td>
<td>0.02%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gender</th>
<th>Deuteranopia</th>
<th>Deuteranomaly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>1.0%</td>
<td>5.0%</td>
</tr>
<tr>
<td>Female</td>
<td>0.01%</td>
<td>0.35%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gender</th>
<th>Tritanopia</th>
<th>Tritanomaly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>.001%</td>
<td>.01%</td>
</tr>
<tr>
<td>Female</td>
<td>.03%</td>
<td>0.01%</td>
</tr>
</tbody>
</table>

source: http://www.color-blindness.com/general/prevalence
Protanopia: Deficient L-cones
Color Blindness: Deuteranopia

Deuteranopia: Deficient M-cones
Trianopia: Deficient S-cones
Spectral Sensitivity Curves
Experience Color Blindness

(a) Normal

(b) Protoanopia

(c) Deuteranopia

(d) Tritanopia

See http://www.vizcheck.com to make the simulated appearances
Ishihara Color Blind Test

(e) ?

(f) spots

(g) 35

(h) spots
End Of the Rainbow Scale

(i) Rainbow

(j) Protoanopia

(k) Deuteranopia

(l) Tritanopia

See http://www.vizcheck.com to make the simulated appearances
End Of the Rainbow Scale

Normal on left; Red-green color blindness on right